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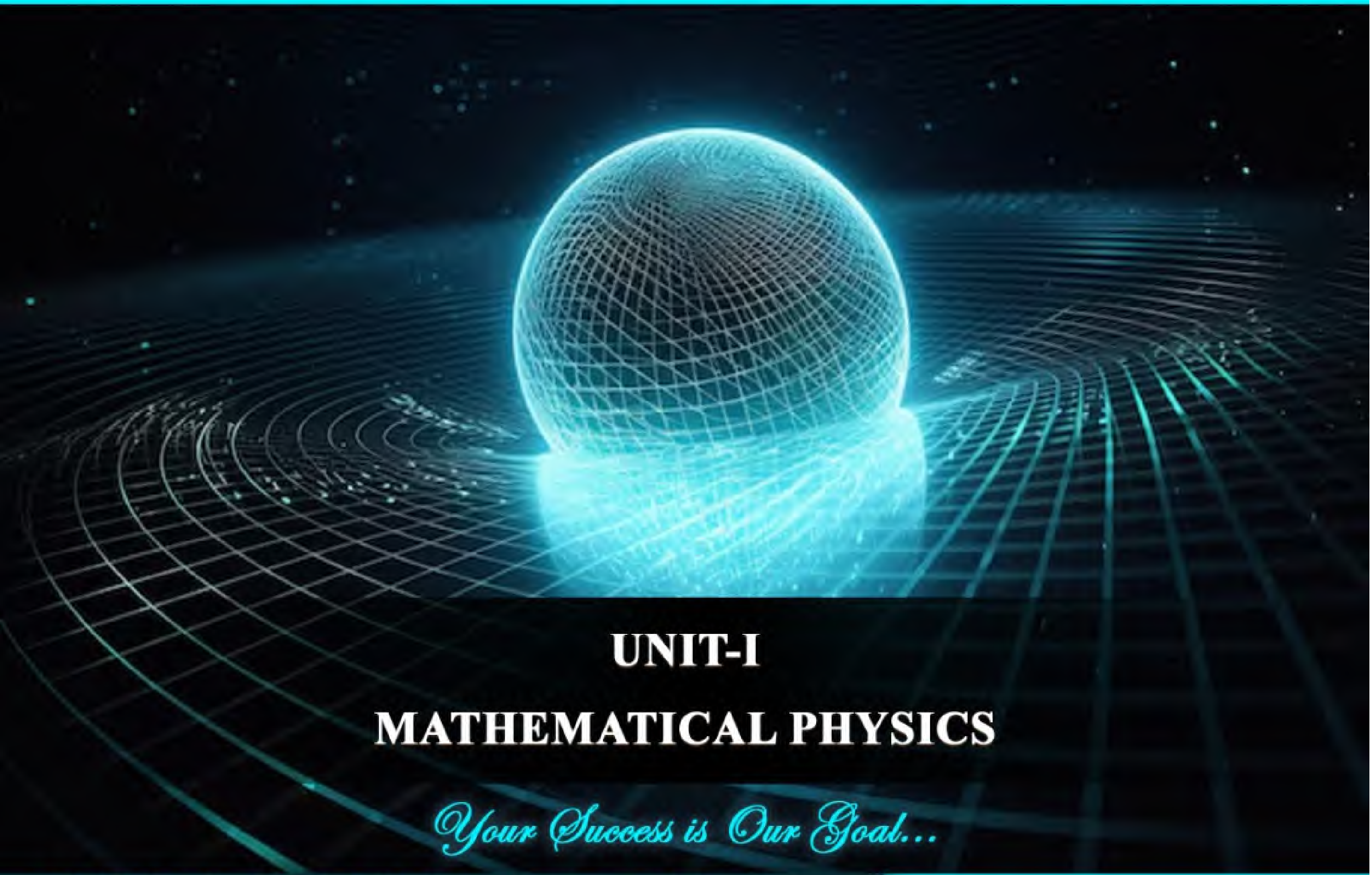
G.O No. 219 New Syllabus

PHYSICS



UNIT-I

MATHEMATICAL PHYSICS

Your Success is Our Goal...

MATHEMATICAL PHYSICS

PG TRB (2025-2026)

UNIT-1

FIRST EDITION

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SPECIAL FUNCTIONS

BETA AND GAMMA FUNCTIONS

UNIT-I MATHEMATICAL PHYSICS

SYLLABUS

Dimensional analysis: Differential equation (ordinary and partial) – order of equation – Expressions for gradient, divergence, curl and Laplacian – vector algebra and vector calculus – Gauss divergence theorem – Green's theorem – Stokes' theorem. Matrix: Cayley – Hamilton theorem, inverse of matrix – Eigen values and Eigen vectors. Polynomials: Hermite, Bessel and Legendre Functions. Special function: Beta and Gamma functions. Probability: Elementary probability theory – Random variables – Binomial – Poisson and Normal distribution. Complex variables: Analytic functions – Singular points – Cauchy's integral theorem and formula -Taylor's and Laurent's expansions, poles, Calculus of residues and evaluation of integrals. Integral transforms: Fourier series and Fourier transform and their properties.

Refference Books :

1) Satya Prakash Mathematics Physics, Edition, Sultan chand & Sons, Reprint 2005
2) H.K. Dass, Dr.Rama Verrma, Mathematical Physics, 8th Revised Edition, S Chand & company Ltd, 2018
3) B.D.Gupta, Mathematical Physics, 4th Edition, Vikas Publishing house (p) Ltd, Reprint 2022
4) B.S Rajput – Mathematical Physics, 28th Edition, pragati Prakashan, Meerut, Reprint 2016
5) A.W. joshi, Matrices and Tensors in Physics, 3rd Edition, New Age International (p) Ltd, 2005
6) K.F Riley and M.P Hobson, Essential Mathematical method for physical sciences, Cambridge University Press 2011

More Reference:

1) PGTRB Previous Year Question Papers
2) UGC NET Previous Year Question Papers
3) SET (State eligibility Test) Previous Year Question Papers (Tamilnadu, Madhya Pradesh, Kerala, Gujarat, Chhattisgarh, Andhra Pradesh, Himachal Pradesh, West Bengal, Uttarakhand, Rajasthan, Maharashtra)

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UNIT – I : MATHEMATICAL PHYSICS

1. 1 DIMENSIONAL ANALYSIS: DIFFERENTIAL EQUATIONS

- A **differential equation** is an equation which contains one or more terms and the derivatives of one variable (i.e., dependent variable) with respect to the other variable (i.e., independent variable) **dy/dx = f(x)**
- Here "x" is an independent variable and "y" is a dependent variable
- For example, dy/dx = 5x
- A differential equation contains derivatives which are either partial derivatives or ordinary derivatives. The derivative represents a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying with respect to the change in another quantity.

1.1.1. Types of Differential Equations

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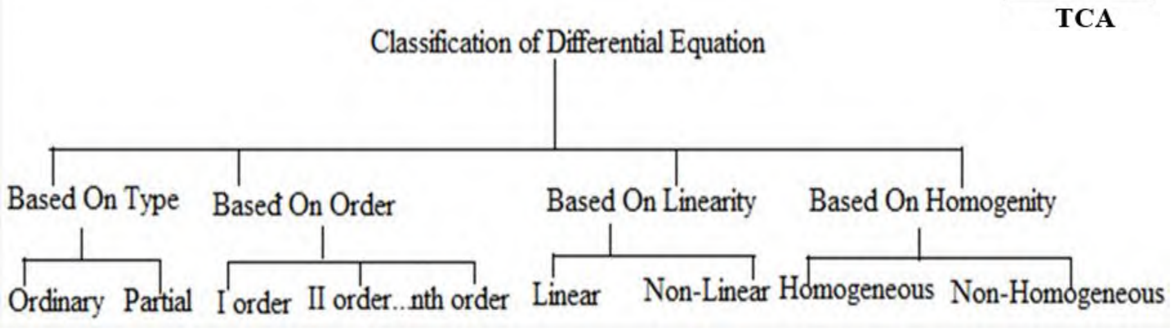
- The below given tree is the general classification of differential equation.



- e.g., $\left(\frac{dy}{dx}\right)^2 + x^2\left(\frac{dy}{dx}\right) = xy$ is an ordinary differential equation while $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\,\partial y} = 0$ is a partial differential equation

1.2.ORDER AND DEGREE OF ORDINARY DIFFERENTIAL EQUATION

Order of the Differential Equation

- The order of the highest order derivative appearing in the differential equation is called the **order** of the differential equation.

Degree of Differential equation:

- If the differential equation is made free of all fractions with regard to powers of all derivatives present, then the exponent of the highest order derivative is called the **degree** of the differential equation.
- e.g., $\left(\frac{d^2y}{dx^2}\right)^{3/2} + \frac{d}{dx} = y + x$ is of order 2 and degree 3 because it can be made into the form

$$\left(\frac{d^2y}{dx^2}\right)^3 = \left(y + x - \frac{dy}{dx}\right)^2$$

- in which the exponent of $\frac{d^2y}{dx^2}$ is 3.
- The degree of a differential equation may not always be defined
- e.g., $\frac{dy}{dx} = \exp\left(\frac{dy}{dx}\right)$ has no degree which is defined.
  - dy/dx + 1 = 0, degree is 1
  - $(y''')^3 + 3y'' + 6y' - 12 = 0$, degree is 3
  - (dy/dx) + cos(dy/dx) = 0; it is not a polynomial equation in y′ and the degree of such a differential equation can not be defined.
- **Note:** Order and degree (if defined) of a differential equation are always positive integers.

1.2.1. General Solution and Particular Solution

- Let $f\left(x, y, y', y'', ..., y^n\right) = 0$ be a differential equation where $y' = \frac{dy}{dx}, \; y'' = \frac{d^2y}{dx^2}, ..., etc$. The solution of the differential equation containing n essential arbitrary constants is called the **general solution** of the differential solution.
- There may also be solutions which has no arbitrary constants and also cannot be obtained from the general solution by any choice of the essential arbitrary constants. Such solutions are called **singular solutions**.
- **Note:**Suppose the differential equation has a solution involving some arbitrary constants. If the set of these arbitrary constants is the minimum, such set then is called essential arbitrary constants.

- e.g., we may be asked to find essential arbitrary constants for the solution to $\frac{d^2y}{dx^2}+x=0$

  $y=A_1\sin\left(x+A_2\right)+B_1\cos\left(x+B_2\right)$

  is a solution, but A_1, A_2, B_1, B_2 are not all essential arbitrary constants, because a smaller set of arbitrary constants will suffice.

  $y=A_1\sin\left(x+A_1\right)+B_1\cos\left(x+B_2\right)$

  $=A_1\left(\sin x\cos A_1+\cos x\sin A_1\right)+=B_1\left(\cos x\cos B_2-\sin x\sin B_2\right)$

  $=\left(A_1\cos A_1-B_1\sin B_2\right)\sin x+\left(A_1\sin A_1+B_1\cos B_2\right)\cos x$

  $y=A\sin x+B\cos x$

- where, $A=A_1\cos A_1-B_1\sin B_2$

  $B=A_1\cos A_1+B_1\cos B_2$

- Thus, the essential arbitrary constants are just A and B.

1.2.2.Linear Differentiation Equation

- A linear differential equation is defined as a differential equation of the type

  $\left(D^n+P_1D^{n-1}+P_2D^{n-2}+...+P_n\right)y=Q$

- where, $D=\frac{d}{dy},\ D^2=\frac{d^2}{dy^2},...,etc$
- and $P_1, P_2, ..., P_n$, Q are functions only of x. If atleast one of $P_1, P_2, ..., P_n$ or Q involves y, it is no longer linear.
- Linear differential equations will be studied separately as first order and higher order. First we deal with first order linear differential equation. The most general form of the linear differential equation of first order is

$$\frac{dy}{dx}+Py=Q$$

- where P and Q are functions of x.

Method of Solution of the Equation $\frac{dy}{dx}+Py=Q$

- Multiply both sides by $e^{\int P\,dx}$,

$$e^{\int P\,dx}\frac{dy}{dx}+Pe^{\int P\,dx}y=Qe^{\int P\,dx}$$

or $$\left(P+\frac{d}{dx}\right)\left(ye^{\int Pdx}\right)+Qe^{\int Pdx}$$

- Integrating with respect to x,

$$ye^{\int P\,dx}=\int Q\,e^{\int P\,dx}dx+c$$

- where c is an arbitrary constant
- **Note:** The factor $e^{\int P\,dx}$ is called an integrating factor

Equations Reducible to Linear Form

- Sometimes, there are differential equations which are not linear, yet by using some suitable transformation, can be brought into the linear form. There are no general rules to choose the transformations. The equations themselves suggest the proper transformations.
- $\frac{dy}{dx}+Py=Qy^n$ is not linear yet it can be brought into the linear form by dividing both sides by y^n Hence

$$y^{-n}\frac{dy}{dx}+y^{1-n}P=Q$$

Let $z=y^{1-n}$

$$\Rightarrow \qquad \frac{dz}{dx}=(1-n)\,y^{-n}\frac{dy}{dx}$$

- The differential equations becomes

$$\frac{1}{(1-n)}\frac{dz}{dx}+Pz=Q$$

$$\Rightarrow \qquad \frac{dz}{dx}+(1-n)Pz=(1-n)Q$$

- Which is clearly a linear equation and can be solved by using the integrating factor $e^{\int(1-n)Pdx}$

EXAMPLE PROBLEMS

1. **Solve x** $\ln x\dfrac{dy}{dx}+y=2\ln x$

Solution:

Dividing by $x\ln x,$ we get

$$\frac{dy}{dx}+\frac{1}{x\ln x}y=\frac{2}{x}$$

$\therefore$ IF = integrating factor = $e^{\int\frac{dx}{x\ln x}}$

$$e^{\ln(\ln x)}=\ln x$$

The solution therefore, is

$$y\ln x=\int\frac{2}{x}\ln x\;dx+C$$

$$y\ln x=(\ln x)^2+C$$

$$\Rightarrow\quad y=\ln x+c(\ln x)^{-1}$$

Where C is an arbitrary constant

2. **Solve** $x\dfrac{dy}{dx}-y=x^2$ **with** $y(1)=1$

Solution:

We have

$$\frac{dy}{dx}-\frac{y}{x}=x$$

which is linear differential equation of first order. Hence,

$$IF=e^{\int-\frac{1}{x}dx}=\frac{1}{x}$$

$\therefore$ solution is $\quad y\cdot\dfrac{1}{x}=\displaystyle\int x\cdot\frac{1}{x}dx+c$

$$\Rightarrow\frac{y}{x}=x+c$$

Now, $\quad y(1)=1$

$$\Rightarrow\quad 1=1+c\Rightarrow c=0$$

Solution:

- Hence, $y=x^2$ is the required solution, which is a parabola

3. Solve $\sin x\dfrac{dy}{dx}+y\cos x=x\sin x$

Solution:

This equation may be arranged into the form

$$\frac{dy}{dx}+y\cot x=x$$

$$\therefore\quad IF=e^{\int \cot x\,dx}=e^{\ln(\sin x)}=\sin x$$

The solution therefore, is

$$y\sin x=\int x\sin x dx+c$$

$$=-x\cos x+\sin x+c$$

$$\Rightarrow (y-1)\sin x+x\cos x=c$$

which is the required solution with c as the arbitrary constant.

This equation may also solved by without the use of integrating factors as

$$\sin x\frac{dy}{dx}+y\cos x=\frac{d}{dx}(y\sin x)$$

Hence, $\sin x\dfrac{dy}{dx}+y\cos x=x\sin x$ can be written as

$$\frac{d}{dx}(y\sin x)=x\sin x$$

On integration, we get

$$y\sin x=\int x\sin x\,dx+c$$

$$=-x\cos x+\sin x+c$$

$$\Rightarrow (y-1)\sin x+x\cos x=c$$

4. Solve $x\dfrac{dy}{dx}-y=(x-1)e^{x}$

Solution:

Dividing by x, we get

$$\frac{dy}{dx}-\frac{y}{x}=\frac{(x-1)e^{x}}{x}$$

$$\therefore IF=e^{-\int\frac{dx}{x}}=e^{-\ln x}=\frac{1}{x}$$

The solution therefore, is $\frac{y}{x} = \int\left(\frac{x-1}{x^2}\right)e^x dx$

$$= \int\left(\frac{1}{x} - \frac{1}{x^2}\right)e^x dx$$

Consider $\int \frac{e^x}{x} dx$

$$\int \frac{e^x}{x} dx = \frac{1}{x}e^x + \int \frac{1}{x^2}e^x dx$$

$$\Rightarrow \qquad \int\left(\frac{1}{x} - \frac{1}{x^2}\right)e^x dx = \frac{e^x}{x}$$

Thus, $\frac{y}{x} = \frac{e^x}{x} + c$ where c is an arbitrary constant

5. Solve $xdy = \left\{y + xy^2\left(1+\ln x\right)\right\}dx = 0$

Solution:

Rearranging $x\,dy - y\,dx = xy^2\left(1+\ln x\right)dx$

Dividing by y^2

$$\frac{x\ dy - y\,dx}{y^2} = x\left(1+\ln x\right)dx$$

$$\Rightarrow \qquad -d\left(\frac{d}{y}\right) = x\left(1+\ln x\right)dx$$

On integration, we get

$$-\frac{x}{y} = \int x\left(1+\ln x\right)dx + c$$

$$= x\left(1+\ln x\right)\frac{x^2}{2} - \int \frac{1}{x}\frac{x^2}{2}dx + c$$

$$\Rightarrow \qquad -\frac{x}{y} = \frac{x^2}{2}\left(1+\ln x\right) - \frac{x^2}{4} + c$$

6. Solve $xdy - \left\{y + xy^3\left(1+\ln x\right)\right\}dx = 0$

Solution: Rearranging

$$x\,dy - y\,dx = xy^3\left(1+\ln x\right)dx$$

Dividing by y^3, we get

$$x\frac{dy}{y^3}-\frac{1}{y^2}dx = x\left(1+\ln x\right)dx$$

Let $\quad \dfrac{1}{y^2}=t$

$$\frac{dt}{dx}=-\frac{2}{y^3}\frac{dy}{dx}$$

$$-\frac{x}{2}\frac{dt}{dx}+t = x\left(1+\ln x\right)$$

$$\Rightarrow \qquad \frac{dt}{dx}+\frac{2t}{x}=-2\left(1+\ln x\right)$$

$$\therefore \qquad IF = e^{\int \frac{2}{x}dx}=e^{2\ln x}=x^2$$

The solution therefore, is

$$tx^2=-2\int x^2\left(1+\ln x\right)dx+c$$

$$=\frac{2x^3}{3}\left(\frac{2}{3}+\ln x\right)+c$$

$$\Rightarrow \qquad -\frac{x^2}{y^2}=\frac{2}{3}x^3\left(\frac{2}{3}+\ln x\right)+c$$

which is the required solution.

7. Solve $x\,dy - y\,dx + 3x^3y\,dx - x^2dx = 0$

Solution: Rearranging

$$x\,dy+\left(3x^3y-y-x^2\right)dx=0$$

$$\Rightarrow \qquad x\;dy-y\,dx=\left(x^2-3x^3y\right)dx$$

$$\Rightarrow \qquad \frac{x\;dy-y\;dx}{x^2}=\left(1-3xy\right)dx$$

$$\Rightarrow \qquad d\left(\frac{y}{x}\right)=\left(1-3xy\right)dx$$

Let $\qquad y = vx$

$$\Rightarrow \qquad dv=\left(1-3vx^2\right)dx$$

$$\Rightarrow \qquad \frac{dv}{dx}+3x^2v=1$$

$$\therefore \qquad IF = e^{\int 3x^2dx}=e^{x^2}$$

The solution therefore, is

$$ve^{x^3} = \int e^{x^3} dx + c$$

$$\Rightarrow \qquad \frac{y}{x} e^{x^3} = \int e^{x^3} \cdot dx + c$$

which is the required solution

1.2.3. Higher Order Linear Differential Equations

- Here, we will discuss linear differential equations of order higher than one. For this, we may conveniently divide into two classes where the coefficients $P_1, P_2, ..., P_n$ are constants and where $P_1, P_2, ..., P_n$ are not constants.

Linear Equations with Constants Coefficients

- Let $y^{(n)} + P_1 y^{(n-1)} + ... + P_n y = Q$ be a linear differential equation.
- The differential equation obtained by putting Q = 0

$$y^{(n)} + P_1 y^{(n-1)} + ... + P_n y = 0 \qquad .... (2.1)$$

- is the associated homogenous equation. The solution of the associated homogenous equation is called the complementary solution. Let $y = y_1, y = y_2, ..., y = y_n$ be solution of the homogenous equation.
- Then, $y = c_1 y_1 + c_2 y_2 + ... + c_n y_n$ is the general solution of the homogenous equation and is the complementary function which we shall write it as CF. If $y = f(x)$ is a solution to the linear differential equation

$$y^{(n)} = P_1 y^{(n-1)} + ... + P_n = Q \qquad .... (2.2)$$

- containing no arbitrary constants, it is called the **particular integral** (Written as PI).
- It can be verified that $y = CF + PI$ is the most general solution to the above linear differential equation and has a essential arbitrary constants.

1.2.4.Method of Finding Complementary Function:

- In the case of linear differential equation with constant coefficients, we assume a solution of the homogenous equation to be of the form $y = e^{mx}$, where m is a constant.

$$\text{Thus, } \left(D^n + P_1 D^{n-1} + P_2 D^{n-2} + ... + P_n\right) y = 0 \qquad .... (2.3)$$

- On substitution, $y = e^{mx}$ becomes

$$m^n + m^{n-1} P_1 + m^{n-2} P_2 + ... + P_n = 0 \qquad .... (2.4)$$

- This is an equation of variable m, degree n and will have n roots, $m_1, m_2, m_3, ..., m_n$. The complementary function CF is given by

- $CF = \sum c_i \; e^{m_i x}$, where c_i is an arbitrary constants.
- Note that when all n roots $m_1, m_2, ..., m_n$ are all distinct then $y = \sum c_i e^{m_i x}$ is the complementary function. When some roots are equal then, we do not have n essential arbitrary constants. Suppose $m_1 = m_2$ and rest are all distinct

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + ... + c_n e^{m_n x}$$

$$= (c_1 + c_2) e^{m_1 x} + c_3 e^{m_3 x} + ... + c_n e^{m_n x}$$

$$= c \, e^{m_1 x} + c_3 \; e^{m_3 x} + ... + c_n e^{m_n x}$$

- which has just n – 1 essential arbitrary constants.
- For repeated roots, we take a different line of attack. In operator form of the differential equation

$$\left(D^n + P_1 D^{n-1} + P_2 D^{n-2} + ... + P_n\right) y = 0$$

- Finding m such that $y = e^{mx}$ is a solution, means we substitute m in place of D and solve for m

$$m^n + P_1 m^{n-1} + P_2 m^{n-2} + ... + P_n = 0$$

- which has n roots.
- So, we may write the operator

$$D^n + P_1 D^{n-1} + P_2 D^{n-2} + ... + P_n = 0 \text{ as}$$

$$(D - m_1)(D - m_2)(D - m_3)...(D - m_n) = 0$$

$$\text{If} \qquad m_1 = m_2$$

$$\text{Then} \qquad (D - m_1)^2 y = 0$$

$$\Rightarrow \quad (D - m_1) z = 0 \text{ where } z = (D - m_1) y$$

$$z = c_1 e^{m_1 x}$$

$$(D - m_1) y = c_1 e^{m_1 x}$$

- The solution for y therefore, is

$$y e^{-m_1 x} = c_1 x + c_2$$

$$\Rightarrow \qquad y = (c_1 x + c_2) e^{m_1 x}$$

- Thus, it is clear that when the roots $m_1, m_2, ..., m_n$ all equal, the corresponding part in the CF is

$$CF = \left(c_1 + c_1 x + c_2 x^2 ....c, \; x^{r-1}\right) e^{m_1 x}$$

EXAMPLE PROBLEMS

1.Solve $\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}+11\frac{dy}{dx}+6y=0$

Solution:

The auxiliary equation is

$$m^3+6m^2+11m+6=0$$

$$(m+1)(m+2)(m+3)=0$$

$\Rightarrow m=-1,-2,-3$. The roots are all distinct, Hence,

$$y=c_1e^{-x}+c_2e^{-2x}+c_3e^{-3x}$$

2. Solve $\frac{d^4y}{dx^4}-4\frac{d^3y}{dx^3}+8\frac{d^2y}{dx^2}-8\frac{dy}{dx}+4y=0$

Solution:

The auxiliary equation is

$$m^4-4m^3+8m^2-8m+4=0$$

Dividing by m^2, we get

$$m^2-4m+8-\frac{8}{m}+\frac{4}{m^2}=0$$

$$\left(m+\frac{2}{m}\right)^2-4m+4-\frac{8}{m}=0$$

$$\Rightarrow \left(m+\frac{2}{m}\right)^2-4\left(m+\frac{2}{m}\right)=-4$$

Let $m+\frac{2}{m}=t$

$$\Rightarrow t^2-4t+4=0$$

$$\Rightarrow t=2,2$$

$$\Rightarrow m+\frac{2}{m}=2$$

$$\Rightarrow m^2-2m+2=0$$

$$\Rightarrow m=1\pm i,1\pm i$$

$$\therefore \quad y=(c_1+c_2x)e^{(1+i)x}+(c_3+c_4x)e^{(1-i)x}$$

$$=e^x\left\{(c_1+c_2x)e^{ix}+(c_3+c_4x)e^{-ix}\right\}$$

$$= e^{x}\left\{(c_1' + c_2'x)\cos x + (c_3' + c_4'x)\sin x\right\}$$

which is the required solution

3. Solve $\left(D^4 + 2n^2D^2 + n^4\right)y = 0$

Solution:

The auxiliary equation is

$$m^4 + 2n^2m^2 + n^4 = 0$$

$$\Rightarrow \left(m^2 + n^2\right)^2 = 0$$

$$\Rightarrow m^2 = -n^2, -n^2$$

$$\Rightarrow m = \pm in, \pm in$$

$$\therefore \; y = (c_1 + xc_2)e^{\ln x} + (c_3 + xc_4)e^{\ln x}$$

$$\Rightarrow y = (c_1' + xc_2')\cos nx + (c_3' + xc_4')\sin nx$$

which is the required solution.

1.2.5.Method of Finding the Particular Integral

- Let us denote the operator

$$D^n + P_1D^{n-1} + P_2D^{n-2} + ... + P_n \text{ by } f(D)$$

- Then, the equation $f(D)y = X$ may be solved for y as

$$y = \frac{1}{f(D)}X$$

- The expression $\frac{1}{f(D)}X$ is defined as a function of x which when operated upon by $f(D)$ gives X. In fact, $f(D)$ is the inverse of $\frac{1}{f(D)}$

Thus, $$\frac{1}{D}X = \int X \; dx$$

- Now, let us factorise $f(D)$. If $m_1, m_2, ..., m_n$ are the roots of the auxiliary equation, then

$$f(D) = (D - m_1)(D - m_2)....(D - m_n)$$

$$\Rightarrow \frac{1}{f(D)}X = \frac{1}{(D - m_1)(D - m_2)....(D - m_n)}X$$

- We may first evaluate $\frac{1}{\left(D-m_n\right)}X$ then evaluate $\frac{1}{\left(D-m_{n-1}\right)}\left\{\frac{1}{D-m_n}X\right\}$, etc.
- successively,

Let $\frac{1}{\left(D-\alpha\right)}X=y$

By definition, $\left(D-\alpha\right)y=X$

$\Rightarrow\ y=e^{\alpha x}\int e^{-\alpha x}X\,dx$

Thus, $\frac{1}{\left(D-\beta\right)\left(D-\alpha\right)}X=\frac{1}{\left(D-\beta\right)}\left\{\frac{1}{\left(D-\alpha\right)}X\right\}$

$$=\frac{1}{\left(D-\beta\right)}\int\left\{e^{\alpha x}\int e^{\alpha x}X\,dx\right\}$$

$$=e^{\beta x}\int e^{-\beta x}\left\{e^{\alpha x}\int e^{-\alpha x}X\ \ dx\right\}dx$$

EXAMPLE PROBLEMS

1. Solve $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{3x}$

Solution:

- First we find the complementary function. Let m be a root of the auxiliary equation,

Then, $m^2-5m+6=0$

$\Rightarrow\ \left(m-2\right)\left(m-3\right)=0$

$\Rightarrow\ m=2,3$

$\therefore\ CF=c_1e^{2x}+c_2e^{3x}$

- Next, we find the particular integral, we have

$f\left(D\right)y=e^{3x}$

$\therefore\ PI=\frac{1}{\left(D-2\right)\left(D-3\right)}e^{3x}$

$=\frac{1}{\left(D-2\right)}e^{3x}\int e^{-3x}e^{3x}dx$

$=e^{2x}\int e^{-2x}x\,e^{3x}dx$

$=e^{2x}\int x\,e^{x}dx$

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New Syllabus

PHYSICS

UNIT-II

CLASSICAL MECHANICS

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CLASSICAL MECHANICS

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UNIT-2

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UNIT-II CLASSICAL MECHANICS

SYLLABUS

Mechanics of particles and systems of particles: Constraints and Generalized coordinates, Law of conservation of Energy, Linear and Angular momentum, Conservative and Non-Conservative systems, Degrees of freedom, Holonomic – Nonholonomic – Scleronomic systems. Lagrangian Formalism: Lagrange equations of motion – D'Alembert's principle – Applications (Simple pendulum, Atwood's machine, Harmonic Oscillator, Electrical circuit). Hamiltonian Formalism: Hamilton's equation of motion – Cyclic co-ordinates – Hamilton's equation from variational principle, Principle of least action, Canonical transformation, Liouville's theorem. Rigid body Dynamics: Euler's angles – Moment of inertia tensor, Euler's equation of motion – Symmetrical top, Special theory of Relativity: Inertial and Non- inertial frames, Lorentz transformation, Lorentz inverse transformation, Length contraction, Time dilation, Mass invariance, Einstein's mass-energy relation.

Refference Books :

1) DR.J.C. Upadhaya, Classical Mechanics, 3rd Revised Edition, Himalaya Publishing House, Mumbai
2) Goldstein and Pearson, Classical Mechanics, 3rd Edition, New International Publishers, 2014
3) Gupta, S.L Kumar and Sharma, Classical Mechanics, 3rd Edition, Pragathi Prakasan, Meerut, 2015
4) Classical Mechanics – Satya Prakash, Kedar Nath & co.
5) Classical Mechanics – K.Sankara Rao
6) Classical Mechanics – G.Aruldhass.

More Reference:

1) PGTRB Previous Year Question Papers
2) UGC NET Previous Year Question Papers
3) SET (State eligibility Test) Previous Year Question Papers (Tamilnadu, Madhya Pradesh, Kerala, Gujarat, Chhattisgarh, Andhra Pradesh, Himachal Pradesh, West Bengal, Uttarakhand, Rajasthan, Maharashtra)

UNIT – II : CLASSICAL MECHANICS

2.1. DEGREES OF FREEDOM (DOF)

- Degrees of freedom (DOF) are the number of independent variables that define the possible positions or motions of a mechanical system in space.
- DOF measurements assume that the mechanism is both rigid and unconstrained, whether it operates in two-dimensional or three-dimensional space.
- The number of degrees of freedom is equal to the total number of independent displacements or aspects of motion.
- The Phase space is **6N dimensional space (Junior PG Assistant 2005-06)**
- Degrees of freedom applies to two types of motion: **translational and rotational**. Translational motion refers to movement that is both linear and non-rotational. Rotational motion is just the opposite.
- An unconstrained, rigid mechanism in two-dimensional space (planar space) can potentially have three independent degrees of freedom: two translational and one rotational.
- The number of independent modes of vibrations of a system of N coupled oscillators with N degrees of freedom is **N (PG Assistant 2014-15)**
- For example, Figure 1 shows a block in a planar space. The block, which can represent any rigid, unconstrained mechanism, can move linearly forward and backward along the x-axis, or it can move left or right along the y-axis.
- Both types of movements are considered translational motion.
- The block can also rotate around the z-axis, which means that it also supports rotational motion.

- However, the block cannot rotate around the x-axis or y-axis, nor can it move linearly up and down the z-axis.

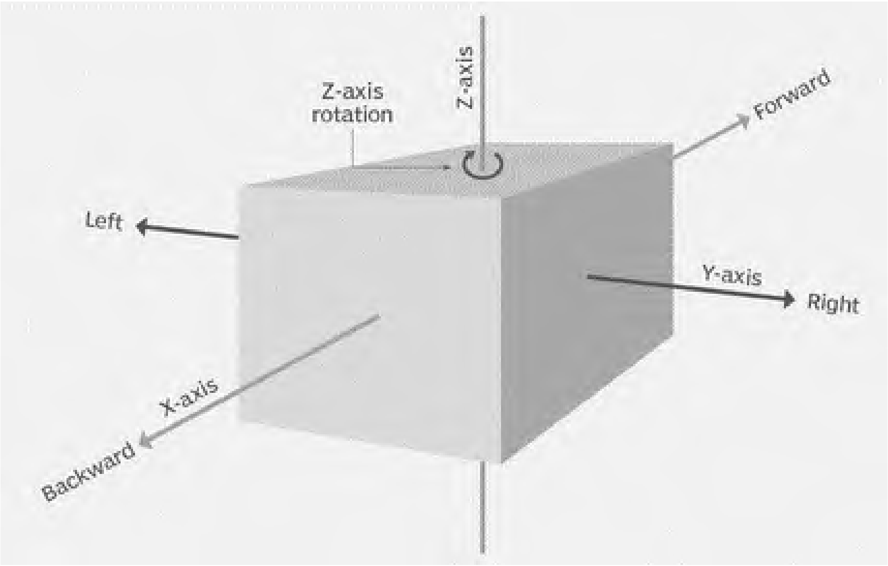

Figure 1. Diagram illustrating a block in a planar (two-dimensional) space and how it supports translational and rotational motion but cannot rotate around x-axis and y-axis or move up and down the z-axis.

- A mechanism in three-dimensional space can support up to six degrees of freedom: three translational and three rotational. Figure 2 shows the same block in a three-dimensional space, but it now includes one more translational motion and two more rotational motions.

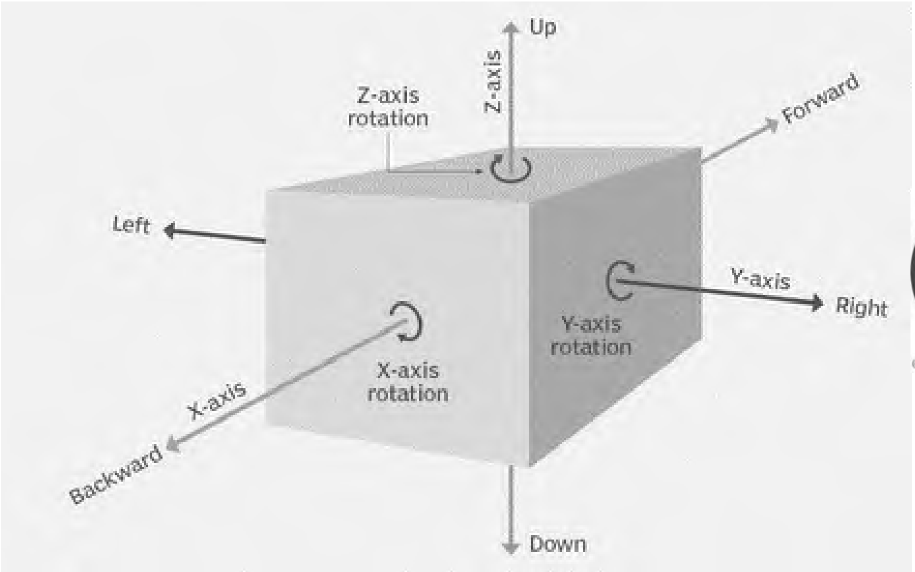

Figure 2. Diagram illustrating how a mechanism in three-dimensional space can support up to six degrees of freedom.

- Because it is in a three-dimensional space, the block can move linearly along all three axes, and it can rotate around all three axes.

- The block can move forward or backward along the x-axis, and it can rotate around the x-axis. In nautical terms, these motions are referred to as surge and roll, respectively.
- The block can move left or right along the y-axis, and it can rotate around the y-axis. In nautical terms, these motions are referred to as sway and pitch, respectively.
- The block can move up or down the z-axis, and it can rotate around the z-axis. In nautical terms, these motions are referred to as heave and yaw, respectively.

➢ Given the use of nautical terms, it's easy to see how a ship can have six degrees of freedom. It exists in a three-dimensional space, and it is capable of all three translational motions and all three rotational motions.

➢ Not all mechanisms in a three-dimensional space have six degrees of freedom. A typical office chair, for example, has four degrees of freedom: three translational and one rotational. The chair can move linearly in any direction, including up and down (to a limited degree), and it can turn in circles around the seat's axis.

Example 1:

➢ If a system is made up of N particles, we need 3N coordinates to specify the positions of all the particles of the system. If a system of N particles are subjected to C constraints (i.e. if some of the particles are connected by C relations), there will be (3N-C) number of independent coordinates only. So, the number of degrees of freedom is (3N-C).

Example 2:

➢ If a point mass is constrained to move in a plane (two dimensions) the number of spatial coordinates necessary to describe its motion is two. So the number of degrees of freedom in this case is 2.

Example 3:

➢ Consider a particle moving on a surface $x^2+ y^2 +z^2= a^2$ In this case the degrees of freedom is 2 though the degrees of freedom in 3-dimensional Cartesian coordinate system is 3.

➢ If the particle is inside the sphere (i.e $x^2+ y^2 +z^2-a^2 <0$), then the number of degrees of freedom is 3.

Example 4:

➢ Consider a system of three free objects. The system has 9 degrees of freedom. If by imposing some constraints the free spaces between the objects are fixed, then the number of degrees of freedom of the system will be 9-3=6. This 6 degrees of freedom can be chosen in any way. For example, the three coordinates of the centre of mass with the 3 angles of their inclinations to a fixed frame of reference.

Example 5:

- In case of a spherical pendulum of string length l, the number of degrees of freedom is 2.
- One can select freely the set of coordinates used to describe the system, keeping in mind that, the number of coordinates minus the number of constraints must give the number of degrees of freedom for that system.

Note:

1. The number of degrees of freedom is independent of the choice of coordinate system.
2. The number of coordinates and number of constraints do not have to be the same for all possible choices.
3. There are freedoms of choices of origin, coordinate system.

2.2. CONSTRAINTS

- Generally, the motion of a particle or system of particles is restricted by one or more conditions. The restrictions on the motion of a system are called constraints and the motion is said to be constrained motion.
- A constrained motion cannot proceed arbitrarily in any manner. For example, a particle motion is restricted to occur only along some specified path, or on a surface (plane or curved) arbitrarily oriented in space. The motion along a specified path is the simplest example of a constrained motion.

2.2.1. Holonomic Constraints and Nonholonomic Constraints

- The constraints that can be expressed in the form $\mathbf{f(x_1, y_1, z_1: x_2, y_2, z_2; x_n, y_n, z_n; t) = 0}$, where time t may occur in case of constraints which may vary with time, are called holonomic and the constraints not expressible
- In this way are termed as Non-holonomic, the motion of the particle placed on the surface of sphere under the action of gravitational force is bound by non-holonomic constraint $\mathbf{(r_2 - a_2) \geq 0}$.

a) Constraint relations which are non-integrable are of nonholonomic type. These are constraints which cannot be fully defined until the full solution of the equation of motion is known.

b) Constraint relations expressed by inequalities are nonholonomic constraints. For an example, particles compelled to stay inside a box.

c) Problems under frictional forces give rise to another type of nonholonomic constraints.

d) If the total mechanical energy of the system remains constant during the constrained motion, the constraint is called conservative while if the mechanical energy is not conserved during the constrained motion, the constraint is called dissipative. In such cases, the forces which cause such restrictions on the motion of the object, do some work and as a result, the total mechanical energy does not remain constant.

e) If at every point on the constraint surface both the forward and backward motions are possible, the constraint relation can be expressed in the form of an equation. In such cases, the constraint is known as bilateral constraint. The general form of the bilateral constraint is given by f (r, r, t) = 0. If on the other hand, the constraint relation can not be expressed as an equation, the constraint is called unilateral. In such cases, forward motion is not possible at some point on the constraint surface. General form of the unilateral constraint is (f r, r, t) ≥ 0

- The number of degrees of freedom for the general motion of a right body is **6 (PG Assistant 2014-15)**

2.2.2.Examples of constraints

Example 1:

- In a rigid body, the distance between any two points r_i and r_j remains unchanged $(r_i - r_j)^2 = c_{ij}^2$. i.e. This is an example of holonomic constraint.

Example 2:

- A deformable body is one whose shape can be changed easily. The motion of such a body is governed by the equation $|r_i, r_j| = f(t)$, r_i, r_j (i#j, i, j=1,2,...) being the position vectors of i-th and j-th particles respectively at time t and f(t) is a function of time t. This constraint relation is of the form f(r,t) =0. As the constraint depends on time so it is rheonomic, as it is independent of velocity r, so it is holonomic and since the constraintrelation is expressed as an equation so this is an example of bilateral constraint.

Example 3:

- For a simple pendulum with fixed string length, the position vector of the bob r with respect to the fixed fulcrum at any time t must satisfy the constraint relation $|r|^2 = l^2$, l being the constant length of the string of the pendulum. This relation is of the form f(r)=0. It is independent of time, so sceleronomic. As it is independent of velocity r, so it is holonomic. Moreover, the constraint relation is expressed as an equation, so this is a bilateral constraint.

Example 4:

- Consider the movement of gas molecules in a spherical container of radius R. Gas molecules are constrained by the walls of the spherical container to move only within the container. If r_i be the position vector of the i-th particle of the gas molecule then the constraint relation is given by $x_i^2+y_i^2+z_i^2\leq R^2$ (origin is at the centre of the sphere). Here the
- rolling (without sliding) on constraint is nonholonomic as the constraint relation is expressed as an inequality.

Example 5:

- Let a rod having length l connect two particles on a plane which move in such a manner that the centre of the rod moves in the direction of the rod. According to the problem, the constraints equations are

$$z_1 = z_2 = 0, \left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2 = l^2 \ \& \ \frac{\dot{x}_1 + \dot{x}_2}{x_1 - x_2} = \frac{\dot{y}_1 + \dot{y}_2}{y_1 - y_2}$$

- where $(x_1,y_1),(x_2,y_2)$ are the coordinates of the end points of the rod. In this case we have non integrable constraint relation. So, this is an example of a nonholonomic constraint.

Example 6:

- A pendulum with a fixed support is an example of scleronomic system whereas a pendulum for which the point of support is given an assigned motion, forms a rheonomic system.

Example 7:

- Let us consider the motion of a circular disc. The disc always remains vertical when a horizontal plane. The constraints on this system are given by $dx=r\sin\theta d\varphi$, $dy=r\cos\theta d\varphi$, φ is the angle of rotation and θ is the angle between the plane of the disc and the yz-plane. Both the constraints are nonholonomic.

Example 8:

- Let a spherical ball rolls on a perfectly rough plane without sliding. The velocity of any point in the rolling body (seen from the fixed frame of reference) is given by $\upsilon = \upsilon_c+\omega x r$ where υ_c is the velocity of the centre of mass and r is the distance between the centre of mass and the point under consideration,ω being the rolling velocity. This is an example of scleronomic, nonholonomic, bilateral conservative constraint.

Example 9:

- Consider the case of an expanding or contracting spherical gas container. The motion of each gas particle at any time t is constrained by the equation given by $|r(t)| \leq R(t)$ where R is the radius of the container which varies with time t, r is measured from the centre of the container. This is an example of rheonomic, holonomic, dissipativeand unilateral constraint.
- A non holonomic constraint can be expressed as **inequality i.e., $r^2-a^2>0$ (Junior PG Assistant 2004-05)**

2.2.3. Scleronomic Constraint and Rheonomic Constraints

- If the constraints are independent of time, they are called as scleronomic but if they contain time explicity, they are termed as rheonomic. A bead sliding on a wire is an example of rheonomous constraint.

2.2.4. Constraint forces

- The forces associated with these restrictions are called forces of constraints or constraint forces. The basic properties of the constraint forces are as follows:
    i. They are very strong so it is difficult for the body, under consideration, to deviate slightly from the prescribed path or surface. The prescribed path or surface is the constraint and the equation of the path or surface is the constraint relation.
    ii. The main effect of the constraint forces is to satisfy the constraint relations.
    iii. Constraint forces are elastic in nature and appear at the point of contact of the surface.

Forces of Constraint

- You should know that the constraints are always related to forces which restrict the motion of the system. These forces are known as "forces of constraint". For example, the reaction force on a sliding particle on the surface of a sphere is the force of constraint.

- If we consider the case of a rigid body, the inertial forces of action and reaction between any two particles are the forces of constraint. In a simple pendulum, the force of constraint is the tension in the string. Similarly, in the case of a bead sliding on the wire is the reaction by the wire exerted on the bead at each point.
- These forces of constraint are elastic in nature and generally appear at the surface of contact because the motion due to external applied forces is slowed down by the contact. Newton has not given any direction to calculate these forces of constraint.

- Generally, the forces of constraint act in a direction perpendicular to the surface of constraints while the motion of the object is parallel to the surface. In such cases, the work done by the forces of constraint is zero. These constraints are known as workless and may be called as ideal constraints.

PROBLEM:

1. The number of degrees of freedom of a rigid body in d space-dimensions is ________.

(a) 2d (b) 6

(c) $d(d+1)/2$ (d) d!

Solution:

(c) The number of degrees of freedom of rigid body in 3-space-dimension

$$3\times3-3=6$$

In $\frac{d(d+1)}{2}=\frac{3(3+1)}{2}=6$

So number of degrees of freedom of a rigid body in d-space is $\frac{d(d+1)}{2}$

2. The number of degrees of freedom for a rigid body which

(a) can move freely in space is 6 (b) has one point fixed is 5

(c) has one point fixed is 3 (d) has two points fixed is 1

Solution:

- (a, b, d) the rigid body is rigidly fixed if we specify any three non collinear points. The first non collinear point has three (3) degrees of freedom. The rigid body can move about the fixed point. Every point of the rigid body moves on the surface of a sphere of constant radius.
- Fixing a second point restricts the motion of all particles of the rigid body to a circular motion in a place. The second point has 2 degrees of freedom. The third fixed point will have only one degree of freedom. Thus, a freely moving rigid body has 3 + 2 + 1= 6 degrees of freedom. If one point is already fixed, two the second fixed point and the third fixed point are needed to specify it completely. This gives 2 + 1 = 3 degrees of freedom.
- If two points are already fixed, the rigid body can turn about the line joining them. Hence, a third point requires only 1 degree of freedom.

3. A rigid body moving freely in space but with one point of the rigid body fixed has degrees of freedom.

(a) 2 (b) 3

(c) 4 (d) 5

Solution: (a)

- Since, one point is fixed all the points of the rigid body can move over the surface of a sphere. So, only two angle coordinates are required to fix its orientation. Hence, the number of degrees of freedom is 2.

4. A particle constrained to move on the inner surface of a parabolic of revolution has degrees of freedom

(a) 1 (b) 2

(c) 3 (d) 4

Solution: (b)

- There is a constraint in the given problem. The equation of the paraboloid of revolution, for example $x^2 + y^2 = az$. This eliminates z coordinate out of x, y, z coordinates leaving only z and y as independent coordinates. So, the degrees of freedom is 2.

2.2.5. Dynamical system

- A system of moving particles form a dynamical system. The set of positions of all the particles is called the configuration of the dynamical system. Constraints impose difficulties in studying the dynamics of a system. The forces of constraints acting on a dynamical system restrict some of the coordinates to vary independently. The resulting equations of motion are not necessarily independent. As a result a set of independent coordinates are required for the description of the configuration of a dynamical system.
- Cartesian coordinates are just fine for describing particles that can move unconstrained throughout space. But when the motion is constrained in some way, another choice of coordinates may be preferable. Thus generalized coordinates help us to overcome such type of problem.
- Suppose a particle is constrained to move in a circle in the xy-plane. Then the constraints are given by $x^2(t)+y^2(t)= R^2$, $z(t)=0$ solution of which gives $y(t)= \pm\surd(R^2-x^2(t)$ which gives an ambiguity in sign. Using polar coordinates $x=r\cos\varphi$, $y= r\sin\varphi$ the constraint simply becomes $r=R$. The unambiguous location of the particle is given by φ. So in such case, it is better to use φ as the coordinate and $\varphi^{\cdot}$ as the velocity.

2.3. GENERALIZED COORDINATES

- It is give to a set of independent coordinates sufficient in number to describe completely the state of configuration of a dynamical system.
- These co-ordinates are denoted as,

$$q_1, q_2, q_3, ...q_k, ...q_n$$

- Where n is the total number of generalized coordinates. In fact, these are the minimum number of co-ordinates needed to describe the motion of the system

For example:

(1) a particle constrained to move on the circumference of a circle. Only one

(2) Two generalized coordinates for a system of N particles, constrained by K equations are $n = 3N - K$

- It is not necessary that these co-ordinates should be rectangular, spherical or cylindrical.
- In fact, quantities like length, (length)^2 , angle, energy or a dimensional quantity may be used as generalized coordinated but they should completely describe the state of the system.

  Further these n generalized co-ordinates are not restricted by any constraint.

  For a system of N particle, if x_i, y_i, z_i are the Cartesian co-ordinates of the ith particles, then these co-ordinates in terms of generalized co-ordinates q_k can be expressed as

$$x_i = x_i\left(q_1, q_2, ...q_k, ..q_n, t\right)$$

$$y_i = y_i\left(q_1, q_2, ...q_k, ..q_n, t\right)$$

$$z_i = z_i\left(q_1, q_2, ...q_k, ..q_n, t\right)$$

or in general, the position vector $r_i\left(x_i, y_i, z_i\right)$ of the ith particle is,

$$r_i = r_i\left(q_1, q_2, q_3, ..., q_k ...q_n, t\right)$$

- The system is said to be rhenomic, when there is an explicit time dependence in some or all of the functions.
- If there is not explicit time dependence, the system is called scleronomic and t is not written in the functional dependence i.e.,. $r_i = r_i\left(q_1, q_2, ...q_k ...q_n\right)$

Some examples of generalized coordinates are as follows:

(i) For a simple pendulum of length l, the corresponding generalized coordinate is the angular displacement θ from the vertical.

(ii) For a particle on the surface of a sphere, the corresponding generalized coordinates are, θ,φ:θ,φ being the spherical polar coordinates.

(iii) Consider a rod lying on a plane surface. The generalized coordinates are x, y,θ where (x,y) are the coordinates of one end of the rod and θ is the angle between x-axis and the rod.

(iv) Consider a lamina lying in a plane. For this case, the generalized coordinates are x,y,θ where (x,y) are the coordinates of the centroid and θ is the angle made by a line fixed in the plane.

(v) The case of a block sliding on an inclined plane, the distance (s) from the top down the plane serves the purpose of generalized coordinate. The relationship between s and the Cartesian coordinates (x,y) (indicating the position of the block) are given by x=scosθ, y=ssinθ; θ being the angle of inclination of the plane with the horizontal.

(vi) Consider a bead sliding on a uniformly rotating wire in a force free plane. If (x,y) denotes the position of the bead and ω be the angular velocity then x= rcosωt, y= rsinωt. Here r is the only generalized coordinate.

2.3.1. Generalized Notations

- We shall now develop the usual notation for displacements, velocity, acceleration, momentum, force, potential energy in terms of generalized co-ordinates. We shall find them useful while deriving Lagrange's equations of motion.

(1) Generalized Displacement: Let us consider a small displacement of an N-particle system defined by changes δr_i in Cartesian co-ordinates $r_i\left(i=1,2,...,n\right)$ with time t held fixed. For the sake of simplicity we may consider an arbitrary virtual displacement δr_i, then since r_i are functions of generalized co-ordinates defined by equation;

$$r_i = r_i\left(q_1, q_2, ..., q_3 N, t\right)$$

We have $*\delta r_i = \sum_{j=1}^{3N} \frac{\partial r_i}{\partial q_j}\delta q_j$ (as $\delta t = 0$)

- We have chosen r_i to represent the 3N co-ordinates $\left(x_1, y_1, z_1, ...z_n\right)$ for notational convenience; each r_i is equivalent to three component co-ordinates x_1, y_1, z_1 and so on. δq_j are called the generalized displacements or virtual arbitrary displacements. If q_j is an angle co-ordinate, δq_j is an angular displacement.

(2) Generalized Velocity: Velocity may be described in terms of time derivative q_j of the generalized co-ordinate q_j, which is then called generalized velocity associated with a particular co-ordinate q_j. We have for an unconstrained system:

$$r_i = r_i\left(q_1, q_2, ...q_3N, t\right)$$

Then $$\dot{r}_i = \sum_{j=1}^{3N} \frac{\partial r_i}{\partial q_j}\dot{q}_j + \frac{\partial r_i}{\partial t} \qquad \text{---------- (16)}$$

- If the N-system contains k constraints, the number of generalized co-ordinates is 3N – k = f and in that case

$$\dot{r}_i = \sum_{j=1}^{f} \frac{\partial r_i}{\partial q_j}\dot{q}_j + \frac{\partial r_i}{\partial t} \qquad \text{---------- (16')}$$

- Note that if the generalized co-ordinate q_j involves both Cartesian and angle co-ordinates, the generalized velocity associated with a Cartesian co-ordinate $\dot{x}$ is just the corresponding linear velocity *x* while generalized velocity with an angular co-ordinate□□ is the corresponding angular velocity. If a generalized co-ordinate has the dimensions of momentum, the generalized velocity will have the dimensions of force and so on.
- From eq. (16), the Cartesian components of velocity $\dot{r}_i$ are seen to be the linear functions of the generalized velocity components no matter how the generalized co-ordinates are defined. This means that it is easy to express velocities in generalized co-ordinates. The term $\partial r_i / \partial t$ appears only when there are constraints depending on time or in some cases where it is convenient to introduce moving co-ordinate axes.

(3) Generalized Acceleration: Components of accelerations are given by differentiating eq. (16) and (16)', as the case may be

- Equation (16) is

$$\dot{r}_i = \sum_{j=1}^{3N} \frac{\partial r_i}{\partial q_j}\dot{q}_j + \frac{\partial r_i}{\partial t}$$

$$X = X\left(x_1, x_2, x_3\right)$$

$$dX = \frac{\partial X}{\partial x_1}dx_1 + \frac{\partial X}{\partial x_2}dx_2 + \frac{\partial X}{\partial x_3}dx_3 = \sum_{j=1}^{3} \frac{\partial X}{\partial x_j}dx_j$$

- Differentiating it again w.r.t time, we get

$$\ddot{r}_i = \frac{d}{dt}\left(\sum_{i=1}^{3N}\frac{\partial r_i}{\partial q_j}\right)\dot{q}_j + \frac{d}{dt}\left(\frac{\partial r_i}{\partial t}\right)$$

$$= \sum_{j=1}^{3N}\frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right)\dot{q}_j + \sum_{j=1}^{3N}\frac{\partial r_i}{\partial q_j}\ddot{q}_j + \frac{d}{dt}\left(\frac{\partial r_i}{\partial t}\right)$$

$$= \sum_{j=1}^{3N}\frac{\partial \dot{r}_i}{\partial q_j}\dot{q}_j + \sum_{j=1}^{3N}\frac{\partial r_i}{\partial q_j}\ddot{q}_j + \frac{\partial \dot{r}_i}{\partial t}$$

- Putting for $\dot{r}_i$ from eq. (16), on changing index j to k, we get

$$\ddot{r}_i = \sum_{j=1}^{3N}\frac{\partial}{\partial q_j}\left(\sum_{k=1}^{3N}\frac{\partial r_i}{\partial q_k}\dot{q}_k + \frac{\partial r_i}{\partial t}\right)\dot{q}_j$$

$$+\sum_{j=1}^{3N}\frac{\partial r_i}{\partial q_j}\ddot{q}_j + \frac{\partial}{\partial t}\left(\sum_{k=1}^{3N}\frac{\partial r_i}{\partial q_k}\dot{q}_k + \frac{\partial r_i}{\partial t}\right)$$

$$= \sum_{j=1}^{3N}\sum_{k=1}^{3N}\frac{\partial^2 r_i}{\partial q_j \partial q_k}\dot{q}_k\dot{q}_j + \sum_{j=1}^{3N}\frac{\partial^2 r_i}{\partial q_j \partial t}\dot{q}_j + \sum_{j=1}^{3N}\frac{\partial r_i}{\partial q_k \partial t}\dot{q}_k + \frac{\partial^2 r_i}{\partial t^2}$$

$$\ddot{r}_i = \sum_{j=1}^{3N}\frac{\partial r_i}{\partial q_j}\ddot{q}_j + \sum_{j=1}^{3N}\sum_{k=1}^{3N}\frac{\partial^2 r_i}{\partial q_j \partial q_k}\dot{q}_j\dot{q}_k + 2\sum_{j=1}^{3N}\frac{\partial^2 r_i}{\partial q_j \partial t}\dot{q}_j + \frac{\partial^2 r_i}{\partial t^2}$$

---------- (17)

- The Cartesian components of acceleration are not linear functions of components of generalized accelerations $\ddot{q}_j$ alone, but depend quadratically and linearly on the generalized velocity components $\dot{q}_j$ as well. However, in the new approach devised by Lagrange, the computation of second derivatives of generalized co-ordinates is not required.

(4) Generalized Momentum: Let us first write down an expression for kinetic energy in terms of generalized velocities. The kinetic energy T of a system of N free particles in terms of Cartesian co-ordinates is

$$T = \sum_{i=1}^{N}\frac{1}{2}m_i\dot{r}_i^2 = \sum_{i=1}^{N}\frac{1}{2}m_i\left(\dot{r}_i \cdot \dot{r}_i\right)$$

---------- (18)

Substituting for $\dot{r}_i$ from equation (16), we obtain

$$T = \sum_{i=1}^{N}\frac{1}{2}m_i\left[\sum_{j=1}^{3N}\frac{\partial r_i}{\partial q_j}\dot{q}_j + \frac{\partial r_i}{\partial t}\right]\cdot\left[\sum_{k=1}^{3N}\frac{\partial r_i}{\partial q_k}\dot{q}_k + \frac{\partial r_i}{\partial t}\right]$$

$$= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{3N}\sum_{k=1}^{3N}m_i\frac{\partial r_i}{\partial q_j}\cdot\frac{\partial r_i}{\partial q_k}\dot{q}_j\dot{q}_k + \frac{1}{2}\sum_{i=1}^{N}m_i\left[\sum_{j=1}^{3N}\frac{\partial r_i}{\partial q_j}\dot{q}_j + \sum_{k=1}^{3N}\frac{\partial r_i}{\partial q_k}\dot{q}_k\right]\cdot\frac{\partial r_i}{\partial t}$$

$$+\frac{1}{2}\sum_{i=1}^{N}m_i\left(\frac{\partial r_i}{\partial t}\right)^2$$

Second term on the right side consists of two sums which are identical so that

$$T=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{3N}\sum_{k=1}^{3N}m_i\frac{\partial r_i}{\partial q_j}\cdot\frac{\partial r_i}{\partial q_k}\dot{q}_j\dot{q}_k+\sum_{i=1}^{N}\sum_{k=1}^{3N}m_i\frac{\partial r_i}{\partial q_k}\dot{q}_k+\frac{\partial r_i}{\partial t}+\frac{1}{2}\sum_{i=1}^{N}m_i\left(\frac{\partial r_i}{\partial t}\right)^2$$

$$=T^{(2)}+T^{(1)}+T^{(0)} \qquad \text{---------- (19)}$$

- Thus, the general kinetic energy in terms of generalized velocities comprises three distinct terms; $T^{(2)}$ contains terms quadratic in generalized velocities ad this fact is indicated by a superscript (2) on T; $T^{(1)}$ containing linear terms and $T^{(0)}$ is independent of generalized velocities $T^{(1)}$ and $T^{(0)}$ will vanish when $\frac{\partial r_i}{\partial t}=0$ i.e., when our defining eq. (13) has no explicit dependence on time i.e., when moving co-ordinate systems are not involved. In that case $T^{(2)}$ survives and contains cross terms also [like $\dot{q}_j\dot{q}_k\left(j\neq k\right)$] unlike the kinetic energy (18) in Cartesian co-ordinates which is purely a quadratic function of linear velcoites containing only squared terms (such a form is known as homogenous quadratic form or a canonical form). If $T^{(2)}$ is free from cross-terms which will happen when $\frac{\partial r_i}{\partial q_j}\cdot\frac{\partial r_i}{\partial q_k}=0$ for $j\neq k,$ the generalized co-ordinate system in $q_j{}'s$ is referred to as an orthogonal system.

- From equation (18) we observe that linear momentum, associated with the linear velocity $\dot{x}_i$ is $m_i\dot{x}_i,$ is given by

$$p_{xi}=\frac{\partial T}{\partial\dot{x}_i}=m_i\dot{x}_i$$

- Then momentum associated with generalized co-ordinate q_k is similarly defined and is called the generalized momentum p_k associated with a co-ordinate q_k :

$$p_k=\frac{\partial T}{\partial\dot{q}_k} \qquad \text{---------- (20)}$$

- p_k need not always have dimensions $\left(MLT^{-1}\right)$ of linear momentum, for, if q_k happens to be an angular co-ordinate, p_k is the corresponding angular momentum $\left(ML^2T^{-1}\right)$. Differentiating eq. (19) w.r.t. $\dot{q}_k$ we get

$$p_k=\sum_{i=1}^{N}\sum_{j=1}^{3N}\frac{1}{2}m_i\frac{\partial r_i}{\partial q_j}\cdot\frac{\partial r_i}{\partial q_k}\dot{q}_j+\sum_{i=1}^{N}m_i\frac{\partial r_i}{\partial q_k}\cdot\frac{\partial r_i}{\partial t} \qquad \text{---------- (21)}$$

- Last term will again be absent if the generalized system is stationary. It is again a linear function of generalized velocities. We shall compute generalized momenta associated with plane polar coordinates (r,θ) below:

$$T = \frac{1}{2}m(\dot{r},\dot{r}) = \frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right), \text{ from equation (18),}$$

and

$$K.E = T = \frac{1}{2}m\left(\frac{\partial r}{\partial r}.\frac{\partial r}{\partial r}\right)\dot{r}\ \dot{r} + \frac{1}{2}m\left(\frac{\partial r}{\partial \theta}\cdot\frac{\partial r}{\partial \theta}\right)\dot{\theta}\ \dot{\theta} + 2\left[\frac{1}{2}m\left(\frac{\partial r}{\partial \theta}\cdot\frac{\partial r}{\partial \theta}\right)\dot{r}\ \dot{\theta}\right]$$

$$= \frac{1}{2}m\left[\left(i\frac{\partial x}{\partial r}+j\frac{\partial y}{\partial r}\right)\cdot\left(i\frac{\partial x}{\partial r}+j\frac{\partial y}{\partial r}\right)\right]\dot{r}\ \dot{r}$$

$$+\frac{1}{2}m\left[\left(i\frac{\partial x}{\partial \theta}+j\frac{\partial y}{\partial \theta}\right)\cdot\left(i\frac{\partial x}{\partial \theta}+j\frac{\partial y}{\partial \theta}\right)\right]\dot{\theta}\ \dot{\theta}$$

$$+2\left[\frac{1}{2}m\left(i\frac{\partial x}{\partial r}+j\frac{\partial y}{\partial r}\right)\cdot\left(i\frac{\partial x}{\partial \theta}+j\frac{\partial y}{\partial \theta}\right)\right]\dot{r}\ \dot{\theta},$$

from equation (19),

- from $x = r\cos\theta$ and $y = r\sin\theta$, we obtain

$$\frac{\partial x}{\partial r} = \cos\theta, \qquad \frac{\partial y}{\partial r} = \sin\theta,$$

$$\frac{\partial y}{\partial \theta} = -r\sin\theta, \qquad \frac{\partial y}{\partial \theta} = r\cos\theta,$$

- Putting them in above equation, we get for kinetic energy in generalized co-ordinates (r,θ) is

$$T = \frac{1}{2}m\left(\cos^2\theta+\sin^2\theta\right)\dot{r}\ \dot{r} + \frac{1}{2}m\left(r^2\sin^2\theta+r^2\cos^2\theta\right)\dot{\theta}\ \dot{\theta} \rightarrow$$

$$+2\left[\frac{1}{2}m\left(-r\cos\theta\sin\theta+r\sin\theta\cos\theta\right)\dot{r}\ \dot{\theta}\right] \qquad \text{---------- (22)}$$

$$= \frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)$$

- which is a familiar expression, The associated generalized momenta are given by equation (20) as

$$p_r = \frac{\partial r}{\partial \dot{r}} = m\dot{r} \qquad \text{(linear momentum)}$$

$$p_\theta = \frac{\partial r}{\partial \dot{\theta}} = mr^2\dot{\theta} \qquad \text{(angular momentum)}$$

- However, the product of any generalized momentum and the associated (also called conjugate) co-ordinate must always have the dimensions of angular momentum or of the dynamic variable 'action' to be defined in chapter 3.

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UNIT-III ELECTROMAGNETIC THEORY

SYLLABUS

Electrostatics: Coulomb's law – Gauss's law and its application. Laplace and Poisson's equations. Magnetostatics: Biot Savart's law – Ampere's law – Magnetic scalar and vector potentials – magnetic susceptibility – Equation of continuity – Displacement current – Maxwell's equations (free space and linear isotropic media) – Electromagnetic waves – Poynting's theorem – Dielectrics: Retarded potentials – Polarization – Radiation from a linear antenna – Transmission lines and Wave guides.

Refference Books :

1) Electro Magnetic Theory and Electro Dynamics - ELECTROMAGNETIC THEORY AND ELECTRODYNAMICS SATYA PRAKASH
2) D. J. Griffiths , Introduction to Electrodynamics, Cambridge University Press, 5th Edition (2023).
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More Reference:

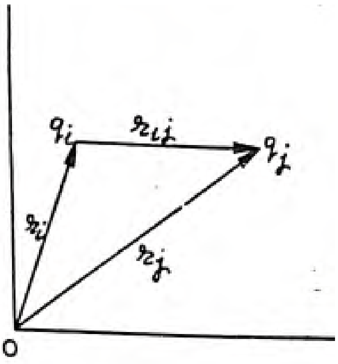
1) PGTRB Previous Year Question Papers
2) UGC NET Previous Year Question Papers
3) SET (State eligibility Test) Previous Year Question Papers (Tamilnadu, Madhya Pradesh, Kerala, Gujarat, Chhattisgarh, Andhra Pradesh, Himachal Pradesh, West Bengal, Uttarakhand, Rajasthan, Maharashtra)

UNIT – III : ELECTROMAGNETIC THEORY

3.1 COULOMBS LAW

- It was found experimentally that there is a force of attraction between two oppositely charged bodies and the force of repulsion between two similarly charged bodies. Coulomb in 1887, gave a law for the force of attraction or repulsion between two electrically charged bodies separated from each other by a definitie distance which is known as Coulomb's law after his name and is given below:
- The force of attraction of repulsion between two electric point charges at a fixed distance apart is directly proportional to the product of two point charges and inversely proportional to the square of the distance between them. The direction of the force is always along a straight line joining the point charges.
- If q_i, and q_j, are the magnitudes of the point charges separated from each other by a distance r. then the force acting on charge q_i, due to charge q_j, is given by

$$\boldsymbol{F_{ij}} = \lambda \frac{q_i q_j}{r_{ij}^2} \widehat{r_{ij}} \qquad \text{--------------(1)}$$



- where $\widehat{r_{ij}}$ is the unit vector directed from q_i to q_j is a constant of proportionality whose numerical value depends upon the medium of separation between the charges and the units in which various physical quantities are expressed. In M.K.S. system force F_{ij} is in Newton, charge q is in Coulomb and constant lambda is written as

$$\lambda = \frac{1}{4\pi\varepsilon}$$

- whereε is known as the (absolute) permittivity of the medium between the charges. Thus in M.K.S. system Coulomb's law may be expressed as

$$F_{ij} = \frac{1}{4\pi\varepsilon}\frac{q_iq_j}{r_{ij}^2}\widehat{r_{ij}}$$

$$F_{ij} = \frac{1}{4\pi\varepsilon}\frac{q_iq_j}{r_{ij}^3}r_{ij}\text{-} \qquad \text{----------- (2a)}$$

- If the medium between the charges in air or vacuum, then $\varepsilon = \varepsilon_0$ the permittivity of free space and has the value

  $\varepsilon_0 = 8.854 \times 10^{-12}$ Coul^2/N-m^2

- So Coulomb's law for force on point charge q_j due to point charge q_i in free space or air is expressed as

  $$F_{ij} = \frac{1}{4\pi\varepsilon}\frac{q_iq_j}{r_{ij}^3}r_{ij} \qquad \text{---------- (2b)}$$

- If the force on q, is to he found, it is only necessary to change every subscript i and j and every j to i. It is important to understand this notation, since in future work it will provide a technique for keeping track of field and source variables.
- It may be noted that Coulomb's law applies to point charges. In the macroscopic sense a point charge is one whose spatial dimensions are very small as compared with any other length occuring in the problem under consideration Coulomb's law is also applicable to the interaction of elementary particles such as protons and electrons and holds for electrostatic repulsion between nuclei at distances greater than 10^{-14}, at relatively smaller distances the short-ranged powerful nuclear forces dominate the picture.
- If more than two point charges are present, the mutual forces are determined by repeated application of eqn. (2). In particular if a system of n charges is considered, the force on jth charge is given by

  $$F_j = \sum F_{ij} = \frac{1}{4\pi\varepsilon_0} q_j \sum_{i\neq j}^{n} \frac{q_i r_{ij}}{r_{ij}^3}\text{-} \qquad \text{------------ (3)}$$

- where the summation is extended over all the charges except the jth one. This is just the principle of superposition for forces which simply states that the force acting on a point charge is the vector sum of the individual forces acting on it.
- The idea of n interacting point charges may be extended to the interaction of a point charge with a continuous charge distribution. To understand the meaning of a continuous charge distribution we recall that electric charge is always an interaction of a point charge with a continuous charge distribution. To understand the meaning of a continuous charge distribution we recall that electric charge is always an integral multiple of a elementury (basic) charge, that of electron. For macroscopic point of view this discreteness of charges causes no difficulty simply because the electronic charge is extremely small in magnitude equal to

1.6091 x 10^{-19}coul. The sinalluess of elementary charge means that macroscopic charges are formed of a very large number of elementary charges, hence any small volume clement in a macroscopic distribution contains a large number of elementary (electronic) charges. Then a charge distribution may be described in terms of a charge density function as the limit of the charge per volume becomes infinitesimal.

- A volume charge density is defined as

$$\rho = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} \qquad \text{----------- (4)}$$

- And a surface charge density is defined as

$$\sigma = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta S} \qquad \text{----------- (5)}$$

- It may be noted that ρ and σ are net charge densities.
- Thus force acting on charge q due to a continuous charge distribution is obtained by replacing summation by integration in (3) viz

$$F_q = \frac{q}{4\pi\varepsilon_0} \int \frac{dq}{r^3} r \qquad \text{---------- (6)}$$

- where dq is the charge on a small element of the charge distribution at distance r from the point charge q and the integration is to be performed overall volume and surface charge distributions. For example if a charge is distributed through a volume V with a density ρ and on the surface S which bounds V with a density σ, then the force exerted by the charge distribution on appoint charge q located at the origin may be expresses as

$$F_q = \frac{q}{4\pi\varepsilon_0} \int_V \frac{r}{r^3} \rho(r) dv + \frac{q}{4\pi\varepsilon_0} \int_S \frac{r}{r^3} \sigma(r) da \qquad \text{----------- (7a)}$$

- But if the point charge is at point r instead of origin, the equation (7a) taken the form

$$F_q = \frac{q}{4\pi\varepsilon_0} \int_V \frac{r - r'}{|r - r'|^3} \rho(r') dv' + \frac{q}{4\pi\varepsilon_0} \int_S \frac{r - r'}{|r - r'|^3} \sigma(r') da' \qquad \text{----------- (7b)}$$

- Here the variable r' is used to locate a point within the charge distribution i.e., it plays the role of source point r_i in (3).
- It is clear that the force on q given by (7) is proportional to q. This observation leads us to introduce a vector field which is independent of q namely the force per unit charge. This vector field is known as the electric field.

MULTIPLE CHOICE QUESTIONS

1. **Coulomb's Law describes the force between:**

(A) Two magnetic poles (B) Two charges at rest

(C) A charge and a magnetic field (D) Two moving charges

2. **The magnitude of the electrostatic force between two point charges q_1 and q_2 separated by a**

(a) $F = k\frac{q_1q_2}{r}$ (b) $F = k\frac{q_1q_2}{r^2}$ (c) $F = k\frac{q_1q_2}{r^3}$ (d) $F = k\frac{q_1q_2}{r^4}$

3. **The direction of the force described by Coulomb's Law is:**

(A) Always attractive

(B) Always repulsive

(C) Along the line joining the two charges

(D) Perpendicular to the line joining the two charges

4. **Coulomb's Law applies to:**

(A) Only static charges (B) Only moving charges

(C) Both static and moving charges (D) Magnetic dipoles

5. **The electrostatic force between two charges is inversely proportional to:**

(A) The distance between them

(B) The square of the distance between them

(C) The product of the charges

(D) The square root of the distance between them

6. **If the distance between two charges is halved, the force between them:**

(A) Doubles (B) Quadruples

(C) Halves (D) Becomes one-fourth

7. **If the charges of two particles are doubled, the force between them becomes:**

(A) Half as large (B) Twice as large

(C) Four times as large (D) Unchanged

8. **The SI unit of electric charge is:**

(A) Ampere (B) Coulomb (C) Newton (D) Volt

9. **Coulomb's Law is consistent with which principle?**

(A) Conservation of charge (B) Newton's third law of motion

(C) Conservation of momentum (D) Both (A) and (B)

10. **Coulomb's Law is valid in:**

(A) Vacuum only (B) Any medium

(C) Conductors only (D) Dielectrics only

3.2 GAUSS LAW AND ITS APPLICATION

- Gauss' law define as , ε_0 times the surface integral of the normal component of E over any closed surface in an electrostatic field which equals the net charge inside the surface. Mathematically,

$$\oint_s \boldsymbol{E.da} = \frac{\boldsymbol{q}}{\boldsymbol{\varepsilon_0}}$$

3.2.1 Derivation of the Gauss' law:

- Soln: Let us consider, a positive charge q is enclosed in a surface of arbitrary shape as shown in Figure. The electric field E at every point of the surface is directed radially outward from the charge, and its magnitude at the point r is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- Over an infinitesimal area da the magnitude and direction of the field remains the same. The component E along the normal is $E_n = E\cos\theta$, where θ is the angle between E and the outward normal to the surface.

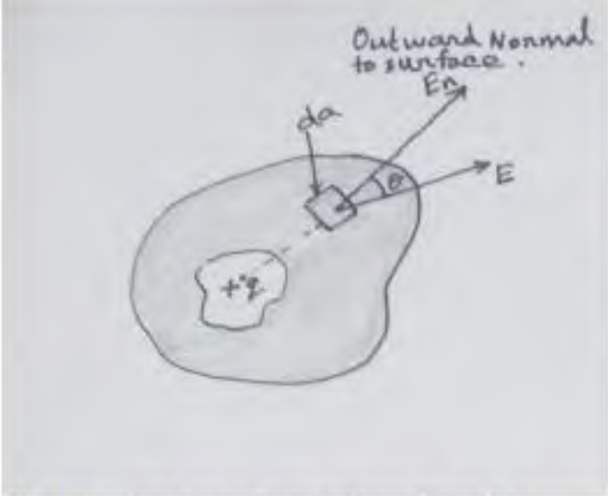


Figure: An imaginary closed surface which encloses a point charge.

- Hence

$$E_n da = E\cos\theta da = \frac{q\cos\theta da}{4\pi\epsilon_0 r^2} \qquad \text{------------ (1)}$$

- **But da cos**θ **is the projection of da along the radius vector r and** $\boldsymbol{\frac{dacos\theta}{r^2} = d\Omega}$
- the element of solid angle subtended by da at the point where q is located. Thus

$$E_n da = \frac{q}{4\pi\varepsilon_0} d\Omega \qquad \text{------------ (2)}$$

- Regardless of the shape or size of the closed surface

$$\int d\Omega = 4\pi \text{Steradians}$$

- Hence integrating over the entire closed surface equation (2) yields

$$\oint_s E_n da = \frac{q}{4\pi\varepsilon_0}\oint d\Omega = \frac{q}{\varepsilon_0} \qquad \text{------------ (3)}$$

- where $\oint_s$ means closed surface integral. Eq. (3) can be put in vector notation if we note that the vector element area da is along the normal. Hence, $E_n da = E \cdot da$. Thus Eq. (3) takes the form $\oint_S E \cdot da = \frac{q}{\varepsilon_0}$
- This is the required gauss' Law.

<u>3.2.2 Application of Gauss' law:</u>

3.2.2.1 Deduction of Coulomb's law by using Gauss' law:

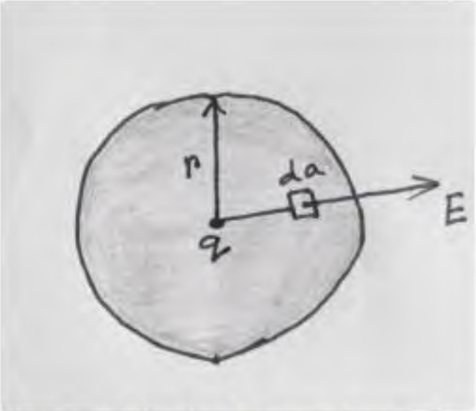


Figure: Spherical Gaussian surface around a point charge.

- Coulomb's law can be deduced from Gauss' law and symmetry considerations. If a point charge q is surrounded by a spherical surface as shown in Figure, then from symmetry consideration the field E is normal to the surface and is constant in magnitude for all point on it. A closed surface so imagined under symmetry consideration will be called a Gaussian surface-a term which we shall often use. In Figure both E and da at any point of the spherical Gaussian surface have direction radially outward. Hence E.da = Eda, and the Gauss' law becomes

$$\oint_S E \cdot da = \oint_S E da = \frac{q}{\epsilon_0}$$

- But since E is constant on the surface, it can be taken outside the integral sign, and consequently $\int_s E \cdot da = \oint_s E da = E \oint_s da = E\left(4\pi r^2\right) = \frac{q}{\epsilon_0}$

Or, $E = \frac{q}{4\pi \epsilon_0 r^2}$

- In vector form $\bar{E} = \frac{q\bar{r}}{4\pi \epsilon_0 r^3}$
- We know that, $F = qE$

- Then the force on charge q_0 at r is

$$F = \frac{q_o q}{4\pi \in_0 r^2}$$

- In vector form $\bar{F} = \frac{q_o q}{4\pi \in_0} \frac{\bar{r}}{r^3}$

3.2.2.2 Electric field due to a charged sphere:

- Let us consider a solid conductive sphere of radius R. If an amount of charge q is placed on the sphere it will be distributed uniformly over the surface of the sphere. No charge can reside in the interior region, because it will disturb the normal distribution of charges in this region of the conductor and hence will create an unbalanced electric

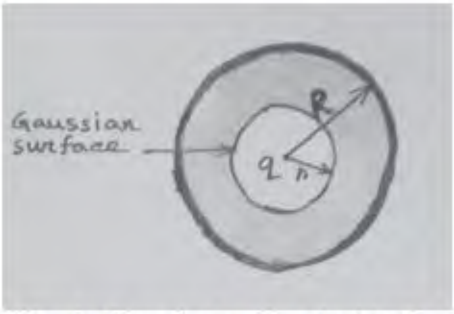


Fig. 1: Gaussian surface inside the charged sphere.

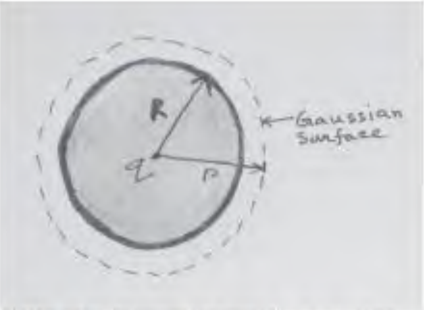


Fig. 2: Gaussian surface outside the charged sphere.

- field causing a current to flow. This cannot happen in the case of electrostatic problems. Therefore, the added charge can only reside on the surface of the conductor. But, the distribution of the charge over the surface must be uniform, otherwise there will exist a component of electric field tangential to the surface thus causing a current to flow on the surface. Again this phenomenon cannot happen in the electrostatic case. The distribution of the charge should be such that it would not create a tangential component of the field so that the field is, at all points on the surface, perpendicular. The above discussion is also true for any conductor of arbitrary shape. Since no tangential component of the field exists on the surface of the conductor, the surface will be an equipotential surface. To find the field at a point r < R of a charged sphere, we imagine a spherical Gaussian surface of radius r concentric with the sphere, Fig. 1. From symmetry and the above argument we see that E can be radial and E is uniform over this Gaussian surface which does not contain any charge. So from Gauss' law,

$$\oint_s \boldsymbol{E.da} = \frac{\boldsymbol{q}}{\boldsymbol{\varepsilon_0}}$$

We get $\oint_S E \cdot da = \oint_S E da = E \oint_S da + E\left(4\pi r^2\right) = 0$

- Hence E = 0 for r < R
- To find the field at r > R, we imagine a similar Gaussian surface passing through that point, Fig, 2, which now includes the charge q. Again E is radial and uniform over this Gaussian surface. So, From Gauss's law,

$$\oint_S E \cdot da = E\left(4\pi r^2\right) = \frac{q}{\in_0}$$

$$\text{or, } E = \frac{q}{4\pi \in_0 r^2} \qquad \text{…….. (1)}$$

- In vector form $\bar{E} = \dfrac{q\bar{r}}{4\pi \in_0 r^3}$ for r > R
- Eq. (1) is equivalent to the field at a point r due to a point charge q. Thus, we may say that while finding the electric field at a point outside of a charged sphere, the charge can be considered to be concentrated at the centre of the sphere and we can use the formula for a point charge.

3.2.2.3 Electric field due to a long charged cylinder:

- Let us consider, a long cylinder of radius a is uniformly charged having a charge λ per unit length. The field E at a point outside the cylinder can be obtained by constructing a Gaussian surface that passes through that point and surrounds an arbitrary length L of the cylinder, shown in Figure. The Gaussian cylindrical surface thus constructed has the symmetry property, and as the charged cylinder is long there is no effect from its ends.

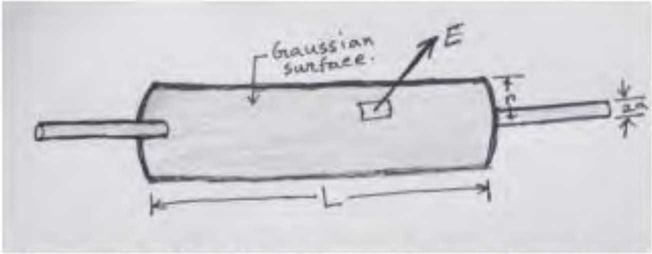


Figure: Cylindrical Gaussian surface for a long charged cylinder

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- The field is everywhere constant on the Gaussian surface, and directed radially away from the axis so that E and da are in the same direction on the curved surface. Hence from Gauss' law,
- Hence from Gauss' law, $\oint_S E \cdot da = \dfrac{q}{\varepsilon_0}$ we get

$$\oint_S E\cdot da = \oint_S E\cdot da = E\left(2\pi rL\right) = \frac{\lambda}{\epsilon_0}L$$

- Where integration is taken over the curved surface

Thus $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$

- In vector notation $\bar{E} = \dfrac{\lambda\bar{r}}{2\pi\epsilon_0 r^2}$ for r > a
- The charged cylinder behaves as if the charge is distributed on a thin wire which passes along the axis of the wire. To find the field in the interior of the charged cylinder, we note that no charge can reside in this region and hence applying Gauss' law we find E = 0 for r < a.

3.2.2.4 Electric field due to a uniformly charge a plane:

- Let us consider, a uniformly charged plane of infinite extent having charged σ per unit area of the surface. An area of the plane can be enclosed by a Gaussian cylinder of the same cross-section as shown in Figure. Now, E is uniform, parallel to da on the ends and perpendicular to da on the curved surface.

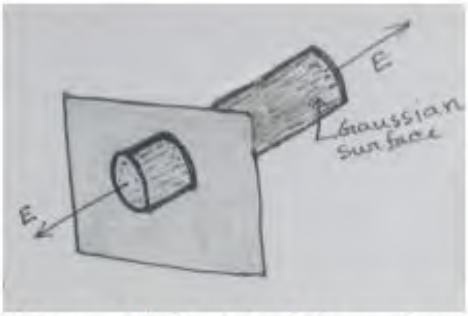


Figure: Cylindrical Gaussian surface for a large uniformly charged plane.

- Hence, we obtain from Gauss' Law

$$\oint_S E\cdot da = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_s \sigma\, da$$

or, $E\Delta A + E\Delta A = \dfrac{1}{\varepsilon_0}\sigma\Delta A$

Hence $E = \dfrac{\sigma}{2\varepsilon_0}$

- This is the required expression for the Electric field due to a uniformly charged plane.

3.2.2.5 Field due to two parallel charged plates:

- Let AB and CD be two parallel plates of very great extent. AB has positive charge and CD has negative. From figure we get, at point P, the field due to plate AB is , pointing right, and that due to plate CD is , pointing right (because the plate CD is charged negatively and E must direct towards it). Therefore, the total field at P is, by the principle of superposition

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

- the direction being from plate AB to plate CD, that is, from positive charge to negative charge.
- The field at Q however, is $E_1 = \frac{\sigma}{2 \in_0}$, towards right due to plate AB and $E_2 = \frac{\sigma}{2 \in_0}$, towards left due to plate CD and the total field is zero. This proves that the field at points outside the plates, such Q and R, vanishes.

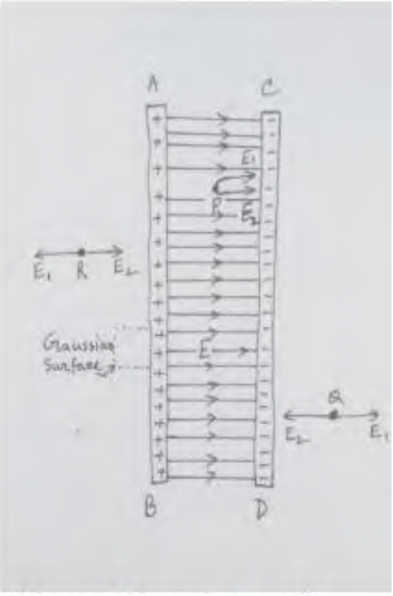


Figure: Gaussian surfaces for two parallel charged plates.

Note: Gauss's Law relates the electric flux and charge (**Pg assistant 2005-2006)**

MULTIPLE CHOICE QUESTIONS

1. **Gauss's Law relates the electric flux through a closed surface to: (PGTRB 2005-06)**

(A) The magnetic flux through the surface

(B) The total charge enclosed within the surface

(C) The electric field on the surface

(D) The potential difference across the surface

2. **The mathematical expression for Gauss's Law is:**

(A) $\nabla \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (B) $\nabla \cdot \vec{E} = -\frac{\rho}{\varepsilon_0}$ (C) $\nabla \cdot \vec{B} = 0$ (D) $\oint \vec{E} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$

3. **The electric flux through a surface is defined as:**

(A) $\vec{E} \cdot \vec{A}$ (B) $\oint \vec{E} \cdot d\vec{A}$ (C) $\int \vec{E} \cdot d\vec{l}$ (D) $\nabla \cdot \vec{E}$

4. **Gauss's Law is valid for:**

(A) Only symmetric charge distributions (B) Any charge distribution

(C) Static charge distributions only (D) Time-varying charge distributions only

5. **Gauss's Law is most useful for calculating electric fields in cases of:**

(A) Arbitrary charge distributions (B) High-frequency electric fields

(C) Symmetric charge distributions (D) None of the above

6. **The electric field inside a uniformly charged spherical shell is:**

(A) Uniform (B) Proportional to the radius

(C) Zero (D) Constant

7. **The electric field outside a uniformly charged spherical shell is equivalent to:**

(A) Zero everywhere (B) The field due to a point charge at the center of the shell

(C) The field due to a uniform sheet of charge (D) A non-uniform electric field

8. **For a long, straight, uniformly charged cylinder, the electric field outside the cylinder varies as:**

(A) $1/r^2$ (B) $1/r$ (C) r^2 (D) Constant

9. **Gauss's Law is consistent with:**

(A) Coulomb's Law (B) Faraday's Law (C) Ampere's Law (D) Lenz's Law

10. **In a parallel plate capacitor with uniform charge density, the electric field between the plates is:**

(A) Zero

(B) Uniform and proportional to the surface charge density

(C) Non-uniform and inversely proportional to the plate separation

(D) Proportional to the square of the surface charge density

3.3 POISSONS AND LAPLACE EQUATIONS

- The Gauss theorem in differential form is

  $divE = \frac{\rho}{\varepsilon_0}$ (in free space) (**PG- ASSISTANT 2003-2004)** ----------- (1)

- Furthermore, in a purely electrostatic field

  E= - grad ϕ ------------ (2)

- Combining (1) and (2) we obtain

  div grad ϕ =ρ/ ε_0 ----------- (3a)

  $\nabla.\nabla\phi = - \rho/ \varepsilon_0$

  $\nabla^2\phi = - \rho/ \varepsilon_0$ ----------- (3b)

- **The operator** $\nabla.\nabla = \nabla^2$is known as the laplacian and the equation (3b) is known as poisons equation. Evidently Laplacian is a pure scalar differential operator and hence Poissons equation is a differential equation. As the operator ∇^2 (Laplacinn) involves differentiation respect to more than one variable. Hence Poisson's equation is partial *differential equation*. If the functional dependence of charge density function $\rho(x,y,z)$and the appropriate boundary conditions are known, then Poisson's equation may be solved for the electrostatic potential ϕ. In order to solve a specific problem we must write ∇^2 in terms of Cartesian co-ordinates (x,y,z) or spherical polar coordinates (r,θ,ϕ)etc. The choice of a particular set of coordinates is arbitrary: but the problem is greatly simplified by the choosing a set compatible with the symmetry of electrostatic problem.
- Laplace's Equation. In charge free region $\rho=0$; hence Poisson's equation charge free region takes be form

  $$\nabla^2\phi = 0 \qquad \text{----------- (4.6)}$$

- This equation as is shown as Laplace's equation.
- We already is known that the scalar potential $\phi$for an arbitrary charge distribution over a given volume is

  $$\phi(r) = \frac{1}{4\pi\varepsilon_0}\cdot\int\frac{\rho(r')}{|r-r'|}dv' \qquad \text{----------- (4.7)}$$

- where $\rho(r')$is the charge density at r' and r is the point of observation.
- The verify that equation (4.7) satisfies Poisson's equation (4.5b), we operator with the Laplacian on both the sides

  $$\nabla^2\phi = \frac{1}{4\pi\varepsilon_0}\nabla^2\int\frac{\rho(r)}{|r-r'|}dv'$$

$$= \frac{1}{4\pi\varepsilon_0}\int \rho(r')\nabla^2\left(\frac{1}{|r-r'|}\right)dv \qquad \text{----------- (4.8)}$$

- Let us now calculate $\nabla^2\left(\frac{1}{|r-r'|}\right)$. For convenience let us translate the origin to r' (i.e., we assume that charge is placed at origin), then $\nabla^2\left(\frac{1}{|r-r'|}\right)$ transforms to $\nabla^2\left(\frac{1}{r}\right)$.
- By direct calculation, we find that

$$\nabla^2\left(\frac{1}{r}\right) = 0 \text{ for } r \neq 0$$

- At r = 0, however the expression is undefined. Hence we must use a limiting process. Since we anticipate something like a Dirac delta function. We integrate $\nabla^2\left(\frac{1}{r}\right)$ over a small volume V containing the origin. Then we use the divergence theorem to obtain a surface integral.

$$\int_v \nabla^2\left(\frac{1}{r}\right)dv = \int_v \nabla\cdot\nabla\left(\frac{1}{r}\right)dv = \int_s \nabla\left(\frac{1}{r}\right)\cdot n\,da$$

$$= \int_s \frac{\partial}{\partial r}\left(\frac{1}{r}\right) r^2 d\Omega = -4\pi$$

- Thus we have

3.3.1 Potential of a Uniform Sphere of Charge

- The use of **Poisson's and Laplace's equations** will be explored for a uniform sphere of charge. In **spherical polar coordinates**, Poisson's equation takes the form:

$$\nabla^2 V = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial\theta^2} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} + \frac{2}{r}\frac{\partial V}{\partial r} + \frac{\cot\theta}{r^2}\frac{\partial V}{\partial\theta} = \frac{-\rho}{\varepsilon_0}$$

- but since there is full spherical symmetry here, the derivatives with respect to θ and φ must be zero, leaving the form

$$\frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\frac{\partial V}{\partial r} = \frac{-\rho}{\varepsilon_0}$$

- Examining first the region outside the sphere, Laplace's law applies.

$\frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\frac{\partial V}{\partial r} = 0$, solution of form $\frac{a}{r} + b$

- Since the **zero of potential** is arbitrary, it is reasonable to choose the zero of potential at infinity, the standard practice with localized charges. This gives the value b=0. Since the sphere of charge will look like a **point charge** at large distances, we may conclude that

$$a = \frac{Q}{4\pi\varepsilon_0} = kQ$$

- so the solution to LaPlace's law outside the sphere is

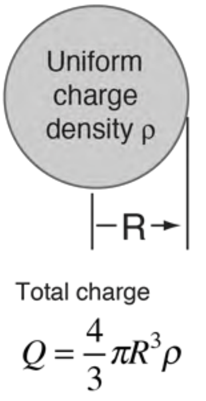
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Now examining the potential inside the sphere, the potential must have a term of order r2 to give a constant on the left side of the equation, so the solution is of the form

$$V = cr^2 + d$$

- Substituting into Poisson's equation gives

$$2c + 4c = \frac{-\rho}{\varepsilon_0} \text{ giving } c = \frac{-\rho}{6\varepsilon_0}$$



- Now to meet the boundary conditions at the surface of the sphere, r=R

$$\frac{-\rho R^2}{6\varepsilon_0} + d = \frac{Q}{4\pi\varepsilon_0 R} \text{ giving } d = \frac{Q}{4\pi\varepsilon_0 R} + \frac{\rho R^2}{6\varepsilon_0}$$

- The full solution for the potential inside the sphere from Poisson's equation is

$$V = \frac{\rho}{6\varepsilon_0}\left[R^2 - r^2\right] + \frac{q}{4\pi\varepsilon_0 R} = \frac{\rho}{6\varepsilon_0}\left[R^2 - r^2\right] + \frac{\rho R^2}{3\varepsilon_0}$$

- Since the **zero of potential** is arbitrary, it is reasonable to choose the zero of potential at infinity, the standard practice with localized charges. This gives the value b=0. Since the sphere of charge will look like a **point charge** at large distances, we may conclude that

$$a = \frac{Q}{4\pi\varepsilon_0} = kQ$$

- so the solution to Laplace's law outside the sphere is

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Now examining the potential inside the sphere, the potential must have a term of order r2 to give a constant on the left side of the equation, so the solution is of the form

$$V = cr^2 + d$$

- Substituting into Poisson's equation gives

$$2c + 4c = \frac{-\rho}{\varepsilon_0} \ giving \ c = \frac{-\rho}{6\varepsilon_0}$$

- Now to meet the boundary conditions at the surface of the sphere, r=R

$$\frac{-\rho R^2}{6\varepsilon_0} + d = \frac{Q}{4\pi\varepsilon_0 R} \text{giving} \, d = \frac{Q}{4\pi\varepsilon_0 R} + \frac{\rho R^2}{6\varepsilon_0}$$

- The full solution for the potential inside the sphere from Poisson's equation is

$$V = \frac{\rho}{6\varepsilon_0}\left[R^2 - r^2\right] + \frac{Q}{4\pi\varepsilon_0 R} = \frac{\rho}{6\varepsilon_0}\left[R^2 - r^2\right] + \frac{\rho R^2}{3\varepsilon_0}$$

MULTIPLE CHOICE QUESTIONS

1. **Poisson's equation reduces to Laplace's equation when:**

   (A) $\rho=\epsilon_0$ (B) $\rho=0$ (C) $\nabla^2V=\mu_0$ (D) $V=0$

2. **Laplace's equation applies in regions where:**

   (A) Charge density is uniform (B) Charge density is zero

   (C) Electric field is zero (D) Magnetic field is constant

3. **Poisson's equation is applicable in regions where:**

   (A) Charge density is zero (B) Charge density is non-zero

   (C) Electric potential is constant (D) Magnetic flux density is uniform

4. **The general form of Laplace's equation is:**

   (A) $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (B) $\nabla^2 V = 0$ (C) $\nabla \times \vec{E} = 0$ (D) $\nabla^2 V = -\frac{\rho}{\epsilon_0}$

5. **Poisson's equation is expressed as:**

   (A) $\nabla^2 V = 0$ (B) $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (C) $\nabla^2 V = -\frac{\rho}{\epsilon_0}$ (D) $\nabla^2 V = -\mu_0 \rho$

6. **A solution to Laplace's equation inside a spherical shell is:**

   (A) A constant value (B) Proportional to r^2

   (C) Zero everywhere (D) A linear function of r

7. **Which of the following boundary conditions is required to solve Laplace's or Poisson's equations?**

   (A) Dirichlet boundary conditions (B) Neumann boundary conditions

   (C) Robin boundary conditions (D) Any of the above, depending on the problem

8. **The potential V(x,y) satisfying Laplace's equation in a 2D region must have:**

   (A) Zero gradient everywhere (B) Constant value everywhere

   (C) Zero second derivative in all directions (D) Zero Laplacian everywhere

9. **Laplace's equation is often used in problems involving:**

   (A) Electrostatics in charge-free regions(B) Magnetostatics in current-free regions

   (C) Steady-state heat conduction (D) All of the above

10. **For a point charge in free space, the solution to Poisson's equation is**

    (A) $V = \frac{\rho}{4\pi\varepsilon_0 r^2}$ (B) $V = \frac{\rho}{4\pi\varepsilon_0 r}$ (C) $V = \frac{q}{4\pi\varepsilon_0 r}$ (D) $V = \frac{\varepsilon_0}{4\pi r}$

11. **In spherical coordinates,the Laplacian for a radially symmetric potentialV(r) is**

    (A) $\nabla^2 V = \frac{d^2V}{dr^2}$ (B) $\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)$ (C) $\nabla^2 V = \frac{1}{r}\frac{dV}{dr}$ (D) $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

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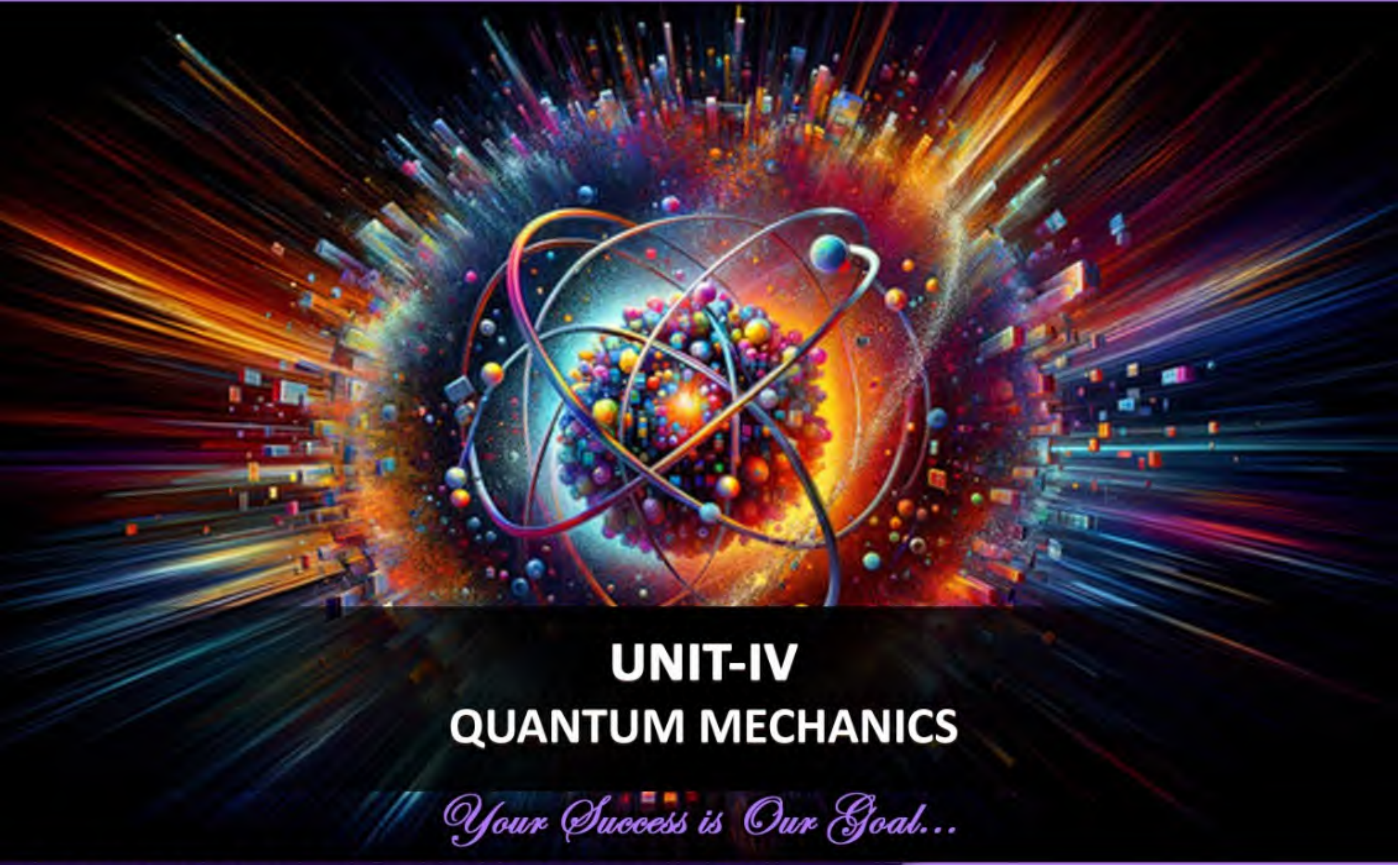

UNIT-IV
QUANTUM MECHANICS

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UNIT-IV QUANTUM MECHANICS

4.14 INTRODUCTION TO THE HYDROGEN ATOM

4.15 STERN-GERLACH EXPERIMENT

4.16 VARIATIONAL PRINCIPLE: APPROXIMATION METHOD IN QUANTUM MECHANICS

4.17 PERTURBATION THEORY: TIME-INDEPENDENT

4.18 TIME DEPENDENT PERTURBATION THEORY

4.19 FERMI'S GOLDEN RULE

4.20 IDENTICAL PARTICLES

4.21 RELATIVISTIC QUANTUM MECHANICS

4.22 PAULI SPIN MATRICES

UNIT-IV QUANTUM MECHANICS

SYLLABUS

Failures of Classical mechanics – Black body radiation – Wave and particle duality – Postulates of Quantum mechanics – Wave function and properties – Expectation values – Heisenberg's uncertainty principle – Schrodinger equations (time – dependent and time- independent). Eigen value problems: Particle in a box (1D and 3D), Particle in a finite potential well & barrier, Tunnelling, Harmonic oscillator. Operators: Ladder operators, Angular momentum operator, Hydrogen atom, spin – Stern Gerlach experiment. Approximation methods: Variational principle, Time independent (1st and 2nd order) degenerate and non-degenerate perturbation theory – Time-Dependent perturbation theory – Fermi's golden rule, Identical particles. Relativistic Quantum Mechanism: Pauli's spin Matrices, Dirac and Klein Gordon equation. Commutators. Scattering theory: Scattering cross-section, Scattering by a central potential, Partial wave analysis, Breit-Wigner formula.

BOOKS TO STUDY:

1) Introduction to Quantum Mechanics: David J. Griffiths.
2) Quantum Physics: H.C. Verma.
3) Quantum mechanics: 500 problems with solutions: G. Aruldhas
4) Principles of Quantum Mechanics: By Ramamurti Shanka
5) Advanced Quantum Mechanics by Sathyamangalam Prakash & Swati Saluja
6) Schaum's Outline of Quantum Mechanics, Second Edition by Yoav Peleg, Reuven Pnini, Elyahu Zaarur

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UNIT – IV: QUANTUM MECHANICS

4.1 FAILURES OF CLASSICAL MECHANICS

- Classical mechanics remains a highly effective theory for everyday situations involving macroscopic objects at low velocities.
- However, it fails to describe phenomena involving extreme velocities, small scales, or strong gravitational fields, and it cannot account for inherently quantum effects.

4.1.1. Inability to Describe High-Velocity Phenomena (Relativistic Speeds)

- Classical mechanics assumes that time and space are absolute, and it does not account for the effects of high velocities approaching the speed of light.
- Albert Einstein's theory of special relativity replaced classical mechanics in this domain, showing that time and space are relative, and introducing concepts such as time dilation and length contraction.
- **Example:** Predictions of classical mechanics fail for particles traveling at speeds close to the speed of light, as in particle accelerators.

4.1.2. Breakdown at Atomic and Subatomic Scales

- Classical mechanics cannot explain the behavior of particles at the atomic or subatomic scale. It predicts deterministic trajectories, which do not align with the probabilistic nature of quantum phenomena.
- Quantum mechanics provides a framework that accurately describes phenomena at small scales, such as wave-particle duality and quantization of energy.
- **Example:** The stability of atoms (e.g., why electrons don't spiral into the nucleus) cannot be explained by classical mechanics but is explained by quantum mechanics.

4.1.3. Inability to Explain Blackbody Radiation

- Classical mechanics, combined with classical electrodynamics, leads to the "ultraviolet catastrophe,"predicting infinite energy in the high-frequency limit of blackbody radiation.
- Max Planck introduced the concept of quantized energy levels, laying the foundation for quantum mechanics.
- ***Example:*** Planck's law of blackbody radiation replaced the Rayleigh-Jeans law derived from classical theory.

4.1.4. Failure to Explain the Photoelectric Effect

- Classical mechanics, coupled with wave theory of light, could not explain why light below a certain frequency cannot eject electrons from a material, regardless of its intensity.
- Einstein proposed that light consists of photons with quantized energy, supporting quantum mechanics.
- ***Example:*** Classical predictions failed to match experimental results for the photoelectric effect.

4.1.5. Chaotic Systems and Sensitivity to Initial Conditions

- Classical mechanics assumes deterministic predictability, but in chaotic systems, tiny errors in initial conditions lead to wildly different outcomes, making long-term predictions impossible.
- Chaos theory, a modern branch of classical mechanics, addresses this but reveals intrinsic limitations in predictability.
- ***Example:*** Weather systems are chaotic, and long-term weather prediction is unreliable despite classical mechanics' equations.

4.1.6.Inadequacy in Describing Strong Gravitational Fields

- Classical mechanics, through Newton's law of gravitation, cannot describe the behavior of objects in strong gravitational fields or space time curvature caused by massive objects.
- General relativity, Einstein's extension of special relativity, replaces Newtonian gravity with the concept of space time curvature.
- ***Example:*** Classical mechanics cannot explain the precise orbit of Mercury, which is resolved by general relativity.

a) Inability to Describe Superfluidity and Superconductivity

- Phenomena like superfluidity and superconductivity, where materials exhibit unusual behaviors at very low temperatures, cannot be explained using classical mechanics.
- ***Example:*** The zero-viscosity flow of superfluid helium defies classical predictions

EXERCISE QUESTIONS

1. **Classical mechanics fails to describe which of the following phenomena?**

(a) Motion of planets

(b) Wave-particle duality of electrons

(c) Projectile motion

(d) Elastic collisions

2. **Assertion (A): Classical mechanics is not valid for describing blackbody radiation.**

Reason (R): Classical mechanics predicts infinite energy at high frequencies, which is unphysical.

(a) Both A and R are true, and R is the correct explanation of A

(b) Both A and R are true, but R is not the correct explanation of A

(c) A is true, but R is false

(d) A is false, but R is true

TCA

3. **The energy density of radiation predicted by classical mechanics is given by the Rayleigh-Jeans law:**

$$u(v,T)=\frac{8\pi v^2 k_B T}{c^3}$$

What happens to u(v,T) as v→∞? How does quantum mechanics resolve this issue?

(a) $u(v,T)\to 0$ as $v\to\infty$; classical mechanics resolves this by introduction energy quantization

(b) $u(v,T)$ diverges as $v\to\infty$; quantum mechanics resolves this using Planck's quantization of energy.

(c) $u(v,T)$ remains constant as $v\to\infty$; this issue is irrelevant to quantum mechanics.

(d) $u(v,T) \to -\infty$ as $v \to \infty$; quantum mechanics uses the uncertainty principle to resolve this.

4. **Classical mechanics fails to describe the stability of atoms because:**

(a) It cannot explain energy quantization in atoms.

(b) It predicts discrete energy levels.

(c) It cannot describe forces between charges.

(d) It does not include the concept of mass.

5. **Which of the following statements is true about the failure of classical mechanics?**

(a) It cannot describe the motion of large objects.

(b) It cannot describe relativistic speeds.

(c) It cannot describe the motion of planets.

(d) It cannot describe projectile motion.

TCA

PRACTICE QUESTIONS

1. **Classical mechanics fails to explain the stability of atoms because:**

(a) Electrons follow deterministic orbits around the nucleus.

(b) Electrons emit continuous radiation while in orbit.

(c) The forces between charged particles are not included in classical mechanics.

(d) Energy quantization is not considered in classical mechanics.

2. **Classical mechanics is unable to describe blackbody radiation because:**

(a) It predicts infinite energy at high frequencies.

(b) It cannot explain the concept of photons.

(c) It does not include wave-particle duality.

(d) It fails to describe the motion of massive objects.

3. **The de Broglie wavelength becomes significant when:**

(a) The mass of the object is very large.

(b) The velocity of the object is very high.

(c) The mass of the object is very small.

(d) The velocity of the object is very low.

4. **Classical mechanics fails at very high velocities because:**

(a) Time dilation and length contraction occur.

(b) The de Broglie wavelength is negligible.

(c) Energy quantization is necessary.

(d) Forces between particles cannot be described.

5. **Which of the following phenomena is not explained by classical mechanics?**

   (a) The motion of planets around the Sun (b) The trajectory of a projectile

   (c) The photoelectric effect (d) The motion of a pendulum

6. **Classical mechanics cannot explain the heat capacity of solids at low temperatures because:**

   (a) The energy of particles is quantized.

   (b) It does not account for atomic vibrations.

   (c) It predicts the Dulong-Petit law for all temperatures.

   (d) It fails to describe elastic collisions.

7. **Which of the following statements is true about classical mechanics?**

   (a) It describes the motion of objects at relativistic speeds.

   (b) It fails to explain the stability of atoms.

   (c) It accounts for the wave-particle duality of light.

   (d) It successfully describes quantum tunneling.

8. **Classical mechanics is valid for which of the following scenarios?**

   (a) Motion of macroscopic objects at low speeds

   (b) Motion of subatomic particles

   (c) Phenomena involving wave-particle duality

   (d) Systems moving close to the speed of light

TCA

9. **The ultraviolet catastrophe refers to:**

   (a) The inability of classical mechanics to describe the photoelectric effect.

   (b) The prediction of infinite energy radiated by a blackbody at high frequencies.

   (c) The failure of classical mechanics to explain atomic stability.

   (d) The collapse of an electron’s orbit into the nucleus.

10. **Which of the following is not a limitation of classical mechanics?**

    (a) Failure to explain quantized energy levels in atoms

    (b) Inability to describe relativistic speeds

    (c) Failure to describe macroscopic object motion at low speeds

    (d) Inability to explain wave-particle duality

4.2 BLACKBODY RADIATION

- Blackbody radiation refers to the electromagnetic radiation emitted by an idealized object (a "blackbody") that absorbs all incident radiation, regardless of wavelength or angle of incidence.
- A blackbody is a perfect absorber and emitter.

4.2.1 Characteristics:

- The radiation depends only on the temperature of the blackbody, not on its material composition.
- Emission spectrum spans all wavelengths but peaks at a wavelength inversely proportional to the temperature.

4.2.2 Key Concepts in Blackbody Radiation

(a) Ideal Blackbody

- A theoretical construct that perfectly absorbs and emits all wavelengths of radiation.
- Real-world examples approximate blackbodies, such as cavity radiators or the Sun.

(b) Emission Spectrum

- At a given temperature, the blackbody emits radiation with a continuous spectrum.
- Energy density increases with temperature and shifts towards shorter wavelengths (higher frequencies).

(c) Laws Governing Blackbody Radiation

a) Planck's Law

$$I\left(\lambda,T\right)=\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}$$

- $I\left(\lambda,T\right)$:Intensity per unit wavelength
- h: Planck's constant $\left(6.626\times10^{-34}\,Js\right)$
- c: Speed of light $\left(3.00\times10^{8}\,m/s\right)$
- λ: Wavelength
- k_B : Boltzmann constant $\left(1.381\times10^{-23}\,J/K\right)$
- T: Absolute temperature (in Kelvin)

Planck's law successfully describes the spectral distribution of blackbody radiation, resolving the ultraviolet catastrophe predicted by classical physics.

b) Stefan-Boltzmann Law

$$P = \sigma T^4$$

- P: Total power radiated per unit area
- σ: Stefan-Boltzmann constant $\left(5.67 \times 10^{-8} W/m^2K^4\right)$
- T: Absolute temperature

The total power radiated by a blackbody is proportional to the fourth power of its absolute temperature.

c) Wien's Displacement Law

$$\lambda_{\max} T = b$$

- $\lambda_{\max}$: Wavelength at which the intensity is maximum
- T: Absolute temperature

- As the temperature increases, the peak wavelength shifts to shorter wavelengths, resulting in a color change (e.g., red-hot to white-hot).

d) Rayleigh-Jeans Law (Classical Prediction)

$$I\left(\lambda, T\right) = \frac{2ck_BT}{\lambda^4}$$

- This law is valid only for long wavelengths and predicts infinite energy as λ approaches zero (ultraviolet catastrophe).

e) Ultraviolet Catastrophe

- Classical physics predicted that the energy radiated at shorter wavelengths (higher frequencies) would become infinite.
- Planck's quantum hypothesis introduced the concept of energy quantization.

4.2.3 Applications of Blackbody Radiation

- **Astrophysics:** Used to estimate the temperature of stars and other celestial bodies (e.g., the Sun's surface temperature is approximated as 5800K).
- **Thermal Imaging:** Blackbody principles are used in infrared cameras to detect heat signatures.
- **Calibration Standards:** Blackbody cavities are used to calibrate radiation measurement instruments.
- **Cosmic Microwave Background (CMB):** The CMB radiation is a near-perfect blackbody spectrum, providing evidence for the Big Bang theory.

EXERCISE QUESTIONS

1. **The spectral radiance of a black body at a given temperature peaks at a wavelength of 500 nm. What is the temperature of the black body?**

   (a) 2900 K (b) 5800 K (c) 4600 K (d) 5000 K

2. **A black body at 1500 K emits radiation with maximum intensity at 1933 nm. If the temperature of the black body is doubled, what will be the wavelength at which the maximum intensity is observed?**

   (a) 1933 nm (b) 966.5 nm (c) 483.3 nm (d) 241.6 nm

3. **The emissive power of a black body is proportional to which of the following?**

   (a) T^2 (b) T^3 (c) T^4 (d) T

4. **A black body emits 100 W of power at a temperature of 300 K. If the temperature is increased to 600 K, what will be the emitted power?**

   (a) 400 W (b) 800 W (c) 1600 W (d) 3200 W

5. **Which of the following expressions represents the total energy emitted per unit area by a black body?**

   (a) σT^4 (b) σT^3 (c) $2\sigma T^4$ (d) σT^2

PRACTICE QUESTIONS

1. **Which law explains the shift in the wavelength of maximum intensity of black body radiation with temperature?**

   (a) Stefan-Boltzmann Law

   (b) Planck's Law

   (c) Wien's Displacement Law

   (d) Rayleigh-Jeans Law

TCA

2. **According to Planck's law, the intensity of radiation emitted by a black body is proportional to which of the following?**

   (a) T^3 (b) v^2 (c) v^3 (d) T^4

3. **At what temperature does a black body radiate with its peak intensity at a wavelength of 1 μm?**

   (a) 500 K (b) 1000 K (c) 3000 K (d) 4000 K

4. **Which of the following plots best represents the variation of energy density with wavelength for a black body at two different temperatures?**
   (a) Energy density shifts left and increases with increasing temperature.
   (b) Energy density shifts right and increases with increasing temperature.
   (c) Energy density remains unchanged, only the peak shifts.
   (d) Energy density decreases with increasing temperature.
5. **If the intensity of radiation from a black body is proportional to v^2, which law is applicable in this limit?**
   (a) Planck's Law
   (b) Wien's Law
   (c) Stefan-Boltzmann Law
   (d) Rayleigh-Jeans Law

4.3 WAVE-PARTICLE DUALITY IN QUANTUM MECHANICS

- ➢ Wave-particle duality is the concept in quantum mechanics that every particle or quantum entity exhibits both wave-like and particle-like properties, depending on how it is observed.
- ➢ This is well explained with the help of Heisenberg's Uncertainity principle.

4.3.1 Heisenberg's Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

- Δx : Uncertainty in position
- Δp : Uncertainty in momentum
- $\hbar$: Reduced Planck's constant $\left(\hbar = h/2\pi\right)$

- ➢ The uncertainty principle highlights that precise measurement of wave-like and particle-like properties (e.g., position and momentum) is fundamentally limited.

4.3.2 Mathematical Formalism

4.3.2.1. Wave-Particle Duality Equation

- Energy: $E = hv$
- Momentum: $p = \frac{h}{\lambda}$

- Relation between energy, momentum, and wave properties bridges wave and particle perspectives.

4.3.2.2. Schrödinger Equation

- The evolution of a quantum system's wavefunction is described by the Schrödinger equation.

- $i\hbar \frac{\partial \psi(x,t)}{\partial t} = \hat{H}\psi(x,t)$

- where $\hat{H}$ is the Hamiltonian operator.

4.3.2.3. Experiments Supporting Wave-Particle Duality

- ***Davisson-Germer Experiment (1927):*** Demonstrated electron diffraction, confirming the wave nature of matter.
- ***C. Jönsson's Double-Slit Experiment (1961):*** Showed interference patterns with single electrons.
- ***Modern Quantum Interference:*** Molecules like C_{60} (buckminsterfullerene) exhibit interference patterns, extending wave-particle duality to larger entities.

4.4 POSTULATES OF QUANTUM MECHANICS

- Quantum mechanics is founded on a set of fundamental postulates that provide a framework for describing and predicting the behavior of quantum systems.
- These postulates formalize the principles of superposition, measurement, and evolution of quantum systems.

4.4.1.Postulate 1: The State of a Quantum System

- The state of a quantum system is completely described by a wave function,
- ψ(x,t), or a state vector, |ψ⟩, in a complex Hilbert space.
- The wavefunction contains all the information about the system.
- The probability density of finding a particle at a position x is given by:

$$P(x) = |\psi(x,t)|^2$$

- The wavefunction must be normalized:

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 \, dx = 1$$

4.4.2.Postulate 2: Observables and Operators

- ➢ Every observable quantity (e.g., position, momentum, energy) is associated with a linear Hermitian operator in the Hilbert space.
- ➢ Observables correspond to measurable physical quantities.
- ➢ Hermitian operators ensure real eigenvalues, which correspond to possible measurement outcomes.

Examples of operators

- Position: $\hat{x} = x$
- Momentum: $\hat{p} = -\hbar\frac{\partial}{\partial x}$
- Energy (Hamiltonian): $\hat{H} = -\frac{h^2}{2m}\nabla^2 + V(x)$

4.4.3.Postulate 3: Measurement and Eigen values

- ➢ The only possible result of a measurement of an observable is one of the eigen values of the corresponding operator.
- ➢ When a measurement is made, the system collapses into an eigen state of the operator associated with the observable.
- ➢ The probability of measuring an eigen value λ_n is given by

$$P(\lambda_n) = \left|\langle\phi_n|\psi\rangle\right|^2$$

- ➢ Where $|\phi_n\rangle$ is the eigen state corresponding to λ_n
- ➢ The expectation value of an observable is:

$$\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle$$

(d) Postulate 4: Time Evolution

- ➢ The time evolution of a quantum system is governed by the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

- ➢ The Hamiltonian represents the total energy of the system.
- ➢ The Schrödinger equation describes how the wavefunction changes with time.
- ➢ For time-independent Hamiltonians, the solution is

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

EXAMPLE QUESTIONS

1. **Which of the following experiments provides direct evidence for the wave nature of particles like electrons?**

   (a) Photoelectric effect (b) Compton effect

   (c) Davisson-Germer experiment (d) Rutherford scattering

2. **According to de Broglie's hypothesis, the wavelength associated with a moving particle is given by:**

   (a) $\lambda = h/p$ (b) $\lambda = h/mv^2$ (c) $\lambda = h/E$ (d) $\lambda = p/h$

3. **In the double-slit experiment, if electrons are used instead of light, what is observed on the detection screen?**

   (a) Two bright spots corresponding to the slits

   (b) A pattern of light and dark fringes

   (c) A single broad spot

   (d) Random scattering of electrons

4. **If the kinetic energy of a particle is doubled, what happens to its de Broglie wavelength?**

   (a) It remains the same

   (b) It is halved

   (c) It is reduced by a factor of $\sqrt{2}$

   (d) It is doubled

5. **Which of the following phenomena cannot be explained by the particle nature of light?**

   (a) Photoelectric effect

   (b) Compton scattering

   (c) Blackbody radiation

   (d) Interference in thin films

6. **An electron and a proton are moving with the same velocity. Which of the following is true about their de Broglie wavelengths?**

   (a) Both have the same wavelength.

   (b) The proton has a longer wavelength.

   (c) The electron has a longer wavelength.

   (d) It depends on their respective energies.

7. **A photon of wavelength 500 nm is incident on a metal surface. The energy of the photon is approximately:**

   (a) 3.98 eV (b) 2.48 eV (c) 4.14 eV (d) 1.65 eV

8. **The uncertainty in position Δx and momentum Δp of a particle are related by:**

   (a) $\Delta x.\Delta p \geq h$ (b) $\Delta x.\Delta p \leq h$ (c) $\Delta x.\Delta p \geq h/4\pi$ (d) $\Delta x.\Delta p \leq h/4\pi$

PRACTICE QUESTIONS

1. **Which of the following experiments provides direct evidence for the wave nature of particles like electrons?**

   (a) photoelectric effect
   (b) Compton effect
   (c) Davisson-Germer experiment
   (d) Rutherford scattering

TCA

2. **In the double-slit experiment, if electrons are used instead of light, what is observed on the detection screen?**

   (a) two bright spots corresponding to the slits
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   (c) a single broad spot
   (d) random scattering of electrons

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   (a) both have the same wavelength

   (b) the proton has a longer wavelength

   (c) the electron has a longer wavelength

   (d) it depends on their respective energies

4.5 WAVE FUNCTION AND ITS PROPERTIES

- The wavefunction, typically denoted by ψ(x,t) in one dimension or ψ(r,t) in three dimensions, is a fundamental concept in quantum mechanics that provides a complete description of a quantum system.
- It is a complex-valued function that encodes information about the quantum state of a particle or system of particles.

4.5.1 Interpretation of the Wave function

4.5.1.1. Probability Amplitude:

- The wavefunction ψ(x,t) itself is not directly observable but represents the probability amplitude of finding a particle at position x and time t.

4.5.1.2. Probability Density:

- The square of the wavefunction's absolute value gives the probability density

$$P(x,t) = |\psi(x,t)|^2$$

Where:

- $P(x,t)dx$: Probability of finding the particle in the interval $[x, x+dx]$ at time t.

In three dimensions:

$$P(r,t) = |\psi(r,t)|^2$$

4.5.1.3.Normalization of the Wavefunction

- The wavefunction must satisfy the normalization condition to ensure the total probability of finding the particle is 1:

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 \, dx = 1$$

In three dimensions:

$$\int |\psi(r,t)|^2 \, d^3r = 1$$

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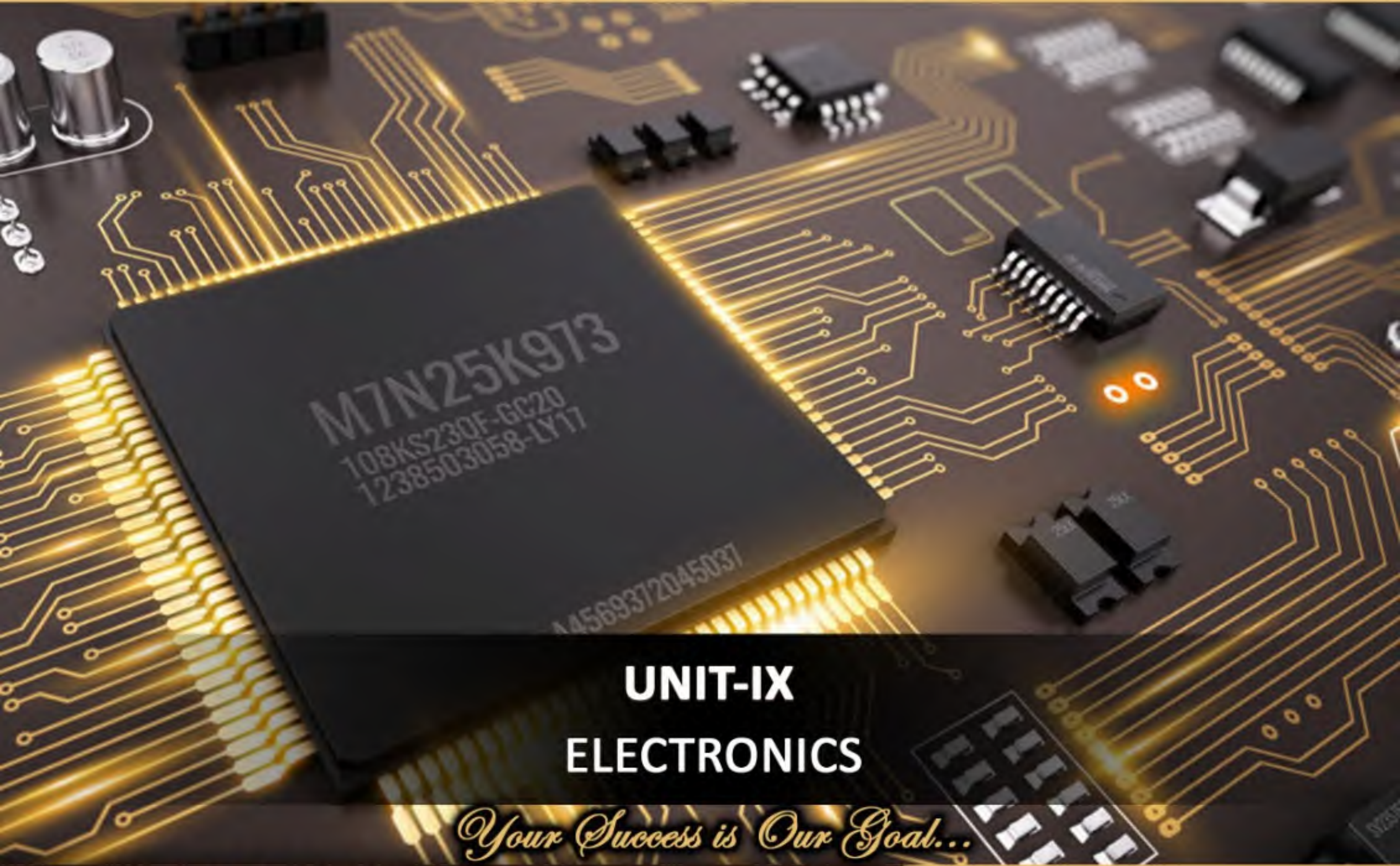

UNIT-IX

ELECTRONICS

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UNIT-IX ELECTRONICS

9.6 ZENER DIODE

9.7 LIGHT EMITTING DIODE (LED)

9.8 BIPOLAR JUNCTION TRANSISTOR [BJT]

9.10 COMMON-BASE (CB) CONFIGURATION

9.11 STATIC CHARACTERISTICS OF TRANSISTOR IN COMMON BASE MODE

9.12 COMMON-EMITTER (CE) CONFIGURATION

9.20 E-MOSFET

9.21 INTEGRATED CIRCUIT

9.22 MONOLITHIC IC PROCESSING

9.23 FABRICATION TECHNOLOGY

9.24 THE 555 TIMER AS MONOSTABLE MULTIVIBRATOR

9.37 DIGITAL INTEGRATED CIRCUITS

9.38 NAND GATE AS A UNIVERSAL GATE

9.39 ELECTRONIC ADDERS

9.40 COMMUNICATION ELECTRONICS

9.41. TYPES OF MODULATION

UNIT-IX ELECTRONICS

SYLLABUS

Semiconducting devices: Diodes – Junction diode – Rectification – Zener diode – Light Emitting Diode. Junction Transistors: common base, common emitter and common collector configurations – Static characteristics – Transistors as amplifier and oscillators – FET, JFET, MOSFET. IC: Fabrication technology, Monolithic IC Processing. 555 Timer, Phase shift, Wien bridge oscillators. Operational Amplifier (IC 741): Op-Amp characteristics, Inverting and Non-inverting Amplifiers, Adder, Subtractor, Differentiator and Integrator. Digital techniques and applications: Flip Flops, Registers – Counters. Digital integrated circuits: Logic gates, NAND and NOR – Universal building blocks – Half and Full adder. Communication Electronics: Modulation and Demodulation (AM, FM Phase), Transmitter and Receiver, Satellite and Fiber optic communication

BOOKS TO STUDY:

1) R,S Sedha , A Text book of Applied Electronics, 2016 3rd Revised Edition, Chand & Company Ltd.
2) B.L. Theraja, Basic Electronics – Soild State, 2012 3rd Revised Edition, & company Ltd, New Delhi 2003.
3) V.K Metha, A Text Book of Applied Electronics, 2019 4th Revised Edition, & Company Ltd, New Delhi 2003.
4) V. Vijayendran, Introduction to Intergrated Electronics (Digital & Analog), S.Viswanathan Printers & Publication.
5) M.Morris Mano, Digital Logic and Computer Design – Pretice Hall of India.

UNIT-IX: ELECTRONICS

SEMICONDUCTING DEVICES

9.1 INTRODUCTION:

9.1.1. Energy band diagram of solids

- In an isolated atom, the electronic energy levels are widely separated and are far apart and the energy of the electron is decided by the orbit in which it revolves around the nucleus.
- However, in the case of a solid, the atoms are closely spaced and hence the electrons in the outermost energy levels of nearby atoms influence each other.
- This changes the nature of the electron motion in a solid from that in an isolated atom to a large extent.
- The valence electrons in an atom are responsible for the bonding nature.
- Let us consider an atom with one electron in the outermost orbit
- It means that the number of valence electrons is one.
- When two such atoms are brought close to each other, the valence orbitals are split up into two.
- Similarly, the unoccupied orbitals of each atom will also split up into two. The electrons have the choice of choosing any one of the orbitals as the energy of both the orbitals is the same. When the third atom of the same element is brought to this system, the valence orbitals of all the three atoms are split into three. The unoccupied orbitals also will split into three.
- In reality, a solid is made up of millions of atoms. When millions of atoms are brought close to each other, the valence orbitals and the unoccupied orbitals are split according to the number of atoms. In this case, the energy levels will be closely spaced and will be difficult to differentiate the orbitals of one atom from the other and they look like a band as shown in Figure 1.
- **This band of very large number of closely spaced energy levels in a very small energy range is known as energy band.**

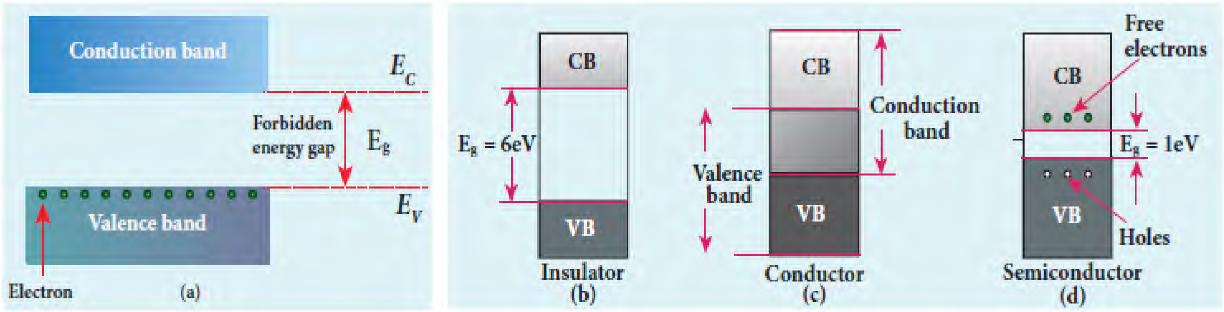

Fig 1 (a) Schematic representation of valence band, conduction band and forbidden energy gap. Energy band structure of (b) Insulator (c) Conductor (d) Semiconductor

- 1(a). E_V represents the maximum energy of the valence band and E_C represents minimum energy of the The energy band formed due to the valence orbitals is called **valence band (VB)** and that formed due to the unoccupied orbitals to which electrons can jump when energised is called the **conduction band (CB).** The energy gap between the valence band and the conduction band is called **forbidden energy gap (E_g).** Electrons cannot exist in the forbidden energy gap.
- A simple pictorial representation of the valence band and conduction band is shown in Figure conduction band. The forbidden energy gap, $E_g = E_C - E_v$. We know that the Coulomb force of attraction between the orbiting electron and the nucleus is inversely proportional to the distance between them. Therefore, the electrons in the orbitals closer to the nucleus are strongly bound to it. Hence, the electrons closer to nucleus require a lot of energy to be excited. The electrons in the valence band are loosely bound to the nucleus and can be easily excited to become free electrons.

9.1.2 Classification of materials

- The classification of solids into insulators, metals, and semiconductors can be explained with the help of the energy band diagram

(i) Insulators

- The energy band structure of insulators is shown in Figure 1(b). The valence band and the conduction band are separated by a large energy gap. The forbidden energy gap is approximately 6 eV in insulators. The gap is very large that electrons from valence band cannot move into conduction band even on the application of strong external electric field or the increase in temperature.
- Therefore, the electrical conduction is not possible as the free electrons are not available for conduction and hence these materials are called insulators. Its resistivity is in the range of 10^{11}–10^{19} Ωm.

(ii) Conductors

- In condutors, the valence band and conduction band overlap **(PG TRB 2001-2002 Q.NO 115)** as shown in Figure 1(c). Hence, electrons can move freely into the conduction band which results in a large number of free electrons available in the conduction band. Therefore, conduction becomes possible even at low temperatures.
- The application of electric field provides sufficient energy to the electrons to drift in a particular direction to constitute a current. For condutors, the resistivity value lies between 10^{-2} Ωm and 10^{-8} Ωm.

(iii) Semiconductors

- In semiconductors, there exists a narrow forbidden energy gap (*Eg*<3*eV)* between the valence band and the conduction band (Figure 1(d)). At a finite temperature, thermal agitations in the solid can break the covalent bond between the atoms (covalent bond is formed due to the sharing of electrons to attain stable electronic configuration).
- This releases some electrons from valence band to conduction band. Since free electrons are small in number, the conductivity of the semiconductors is not as high as that of the conductors. The resistivity value of semiconductors is from 10^{-5} Ωm to 10^{6} Ωm.
- When the temperature is increased further, more number of electrons are promoted to the conduction band and they increase the conduction. Thus, we can say that the electrical conduction increases with the increase in temperature.
- In other words, resistance decreases with increase in temperature. Hence, semiconductors are said to have negative temperature coefficient of resistance. The most important commonly used elemental semiconducting materials are silicon (Si) and germanium (Ge). The values of forbidden energy gap for Si and Ge at room temperature are 1.1 eV and 0.7 eV respectively.

EXERCISE QUESTIONS

1. When an electron jumps from higher orbit to a lower orbit, it ____________ energy.
(A) absorbs (B) emits
(C) sometimes emits, sometimes absorbs (D) none of the above
2. A semiconductor has _________________ band.
(A) almost empty valence (B) almost empty conduction
(C) almost full conduction (D) none of the above

3. The electrons in the conduction band are known as ________________

   (A) bound electrons　　(B) valence electrons

   (C) free electrons　　(D) none of the above

4. In insulators, the energy gap between valence and conduction bands is _________

   (A) very large　　(B) zero

   (C) very small　　(D) none of the above

5. In a conductor, the energy gap between valence and conduction bands is ________

   (A) large　　(B) very large

   (C) very small　　(D) none of the above

6. In a semiconductor, the energy gap between valence and conduction bands is about

   (A) 15 eV　　(B) 100 eV　　(C) 50 eV　　(D) 1eV

7. The energy gap between valence and conduction bands in insulators is about _____

   (A) 15 eV　　(B) 1.5 eV　　(C) zero　　(D) 0.5eV

9.2 TYPES OF SEMICONDUCTRORS

9.2.1 Intrinsic semiconductors:

- **A semiconductor in its pure form without any impurity is called an intrinsic semiconductor**. Here, impurity means presence of any other foreign atom in the crystal lattice. The silicon lattice is shown in Figure 2(a). Each silicon atom has four electrons in the outermost orbit and is covalently bonded with four neighbouring atoms to form the lattice. The band diagram for this case is shown in Figure 2(b).

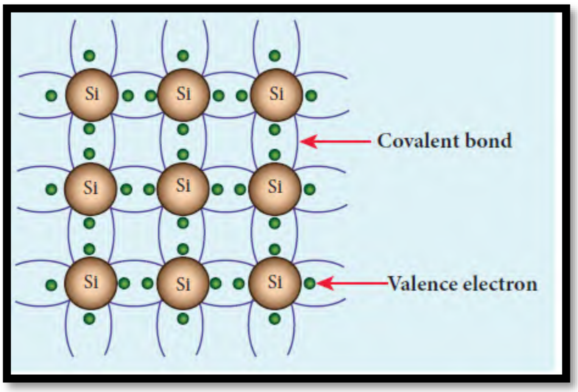

Fig 2 (a) Two dimensional crystal lattice of silicon

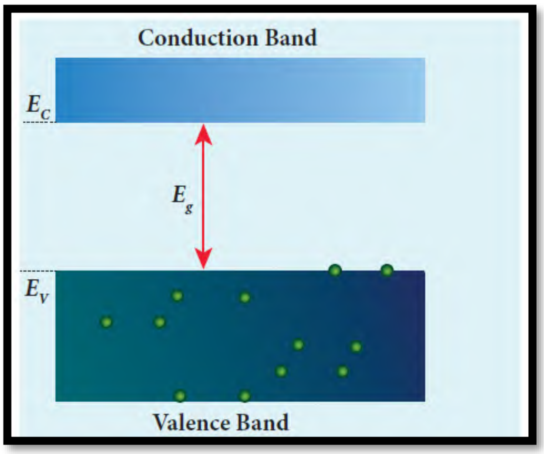

Fig 2(b) Valence band and conduction band of intrinsic semiconductor

- A small increase in temperature is sufficient enough to break some of the covalent bonds and release the electrons free from the lattice (fig 3(a)). As a result, some states in the valence band become empty and the same number of states in the conduction band will be occupied by electrons as shown in Figure 3(b).
- The vacancies produced in the valence band are called holes. As the holes are deficiency of electrons, they are treated to possess positive charges. Hence, electrons and holes are the two charge carriers in semiconductors.
- In intrinsic semiconductors, the number of electrons in the conduction band is equal to the number of holes in the valence band. The electrical conduction is due to the electrons in the conduction band and holes in the valence band. The corresponding currents are represented as I_e and I_h respectively.

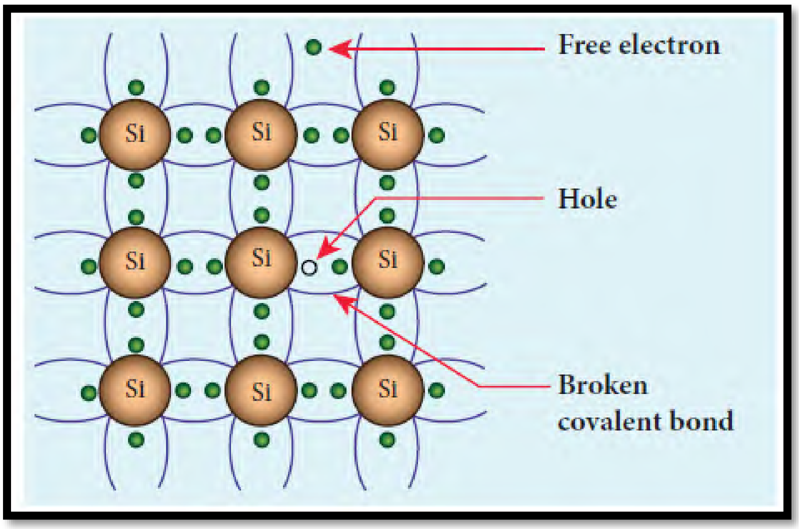

Fig 3 (a) The presence of free electron, hole and broken covalent bond in the intrinsic silicon crystal

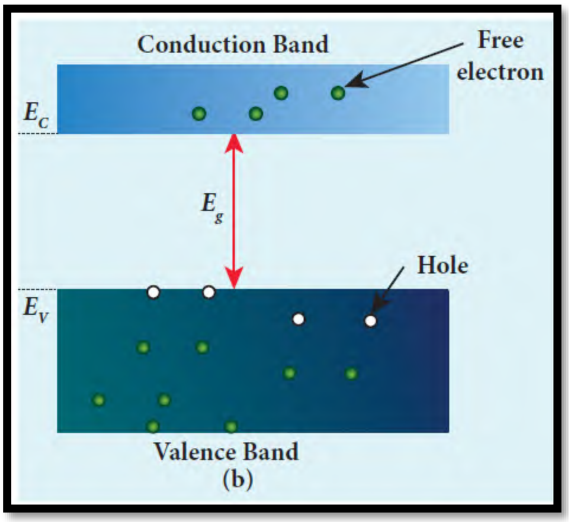

Fig 3 (b) Presence of electrons in the conduction band and holes in the valence band at room temperature.

- The total current *I* is always the sum of the electron current and the hole current. That is, $I = I_e + I_h$. An intrinsic semiconductor behaves like an insulator at 0 K. The increase in temperature increases the number of charge carriers (electrons and holes).
- The schematic diagram of the intrinsic semiconductor in band diagram is shown in Figure 3(b). **The intrinsic carrier concentration is the number of electrons in the conduction band or the number of holes in the valence band in an intrinsic semiconductor.**

9.2.2 Extrinsic semiconductors:

- The carrier concentration in an intrinsic semiconductor is not sufficient enough to develop efficient electronic devices. Another way of increasing the carrier concentration in an intrinsic semiconductor is by adding impurity atoms.
- **The process of adding impurities to the intrinsic semiconductor is called doping.** It increases the concentration of charge carriers (electrons and holes) in the semiconductor and in turn, its electrical conductivity. The impurity atoms are called dopants and its order is approximately 100 ppm (parts per million).
- On the basis of the type of impurity added, extrinsic semiconductors are classified into:
  - (i) *n*-type semiconductor
  - (ii) *p*-type semiconductor

(i) *n*-type semiconductor

- A *n*-type semiconductor is obtained by doping a pure silicon (or germanium) crystal with pentavalent impurity atoms (from V group of periodic table) such as phosphorus, **arsenic (UG TRB 2023 Q.NO 175)** and antimony as shown in Figure 4(a).
- The dopant has five valence electrons while the silicon atom has four valence electrons. During the process of doping, a few of the silicon atoms are replaced by pentavalent dopants.

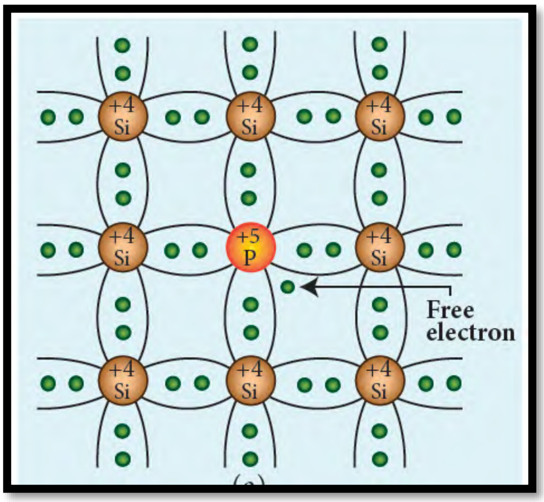

Fig 4 n-type extrinsic semiconductor: (a) Free electron which is loosely attached to the lattice

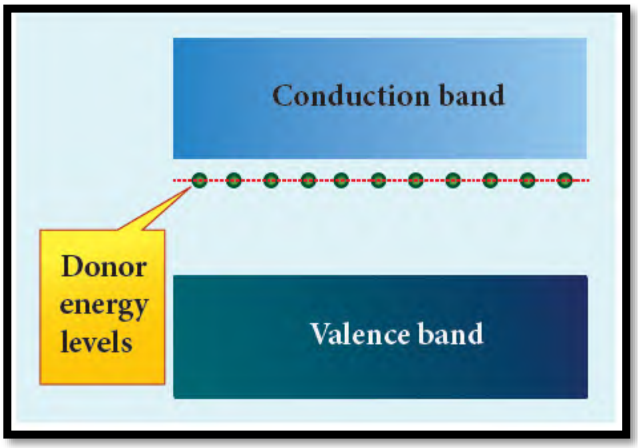

Fig 4 (b) Representation of donor energy level

- Four of the five valence electrons of the impurity atom form covalent bonds with four silicon atoms. The fifth valence electron of the impurity atom is loosely attached with the nucleus as it is not used in the formation of the covalent bond.
- The energy level of the loosely attached fifth electron from the dopant is found just below the conduction band edge and is called the donor energy level as shown in Figure 4(b). At room temperature, these electrons can easily move to the conduction band with the absorption of thermal energy. It is shown in the

Figure 5. Besides, an external electric field also can set free the loosely bound electrons and lead to conduction.

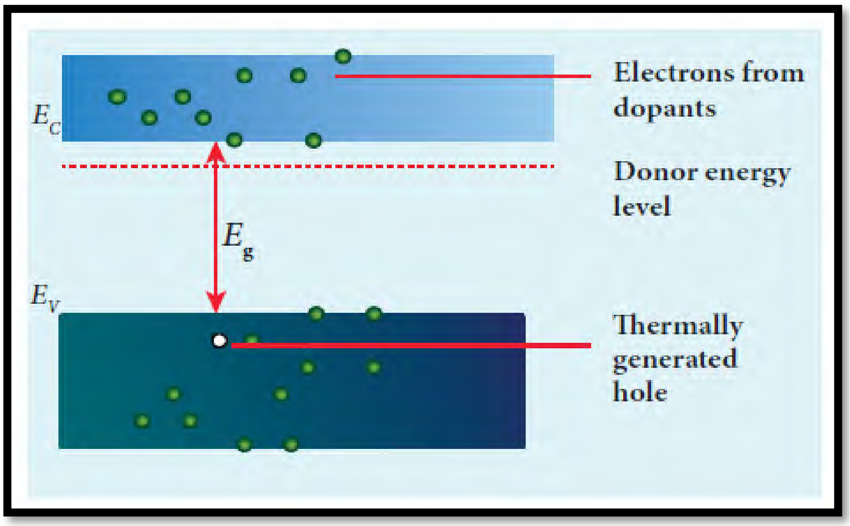

Fig 5: Thermally generated holes in the valence band and the free electrons generated by the dopants in the conduction band (*n*-type semiconductor)

- ❖ It is important to note that the energy required for an electron to jump from the valence band to the conduction band in an intrinsic semiconductor is 0.7 eV for Ge and 1.1 eV for Si, while the energy required to set free a donor electron is only 0.01 eV for Ge and 0.05 eV for Si.
- ❖ The V group pentavalent impurity atoms donate electrons to the conduction band and are called donor impurities. Therefore, each impurity atom provides one extra electron to the conduction band in addition to the thermally generated electrons. These thermally generated electrons leave holes in valence band.
- ❖ Hence, the majority carriers of current in an *n*-type semiconductor are electrons and the minority carriers are holes. Such a semiconductor doped with a pentavalent impurity is called an *n*-type semiconductor.

(ii) *p*-type semiconductor

- ❖ In *p*-type semiconductor, **trivalent impurity atoms (PG TRB 2003-2004 Q.NO 10)** (from III group of periodic table) such as boron, aluminium, gallium and indium are added to the silicon (or germanium) crystal. The dopant with three valence electrons can form three covalent bonds with three silicon atoms.
- ❖ Of the four covalent bonds, three bonds are complete and the remaining one bond is incomplete with one electron. This electron vacancy present in the fourth covalent bond is represented as a hole.
- ❖ To make complete covalent bonding with all four neighbouring atoms, the dopant is in need of one more electron. These dopants can accept electrons from the neighbouring atoms. Therefore, this impurity is called an acceptor impurity. The

energy level of the hole created by each impurity atom is just above the valence band and is called the acceptor energy level, as shown in Figure 6(b).

- For each acceptor atom, there will be a hole in the valence band; this is in addition to the holes left by the thermally generated electrons. In such an extrinsic semiconductor, holes are the majority carriers and thermally generated electrons are minority carriers as shown in Figure 7. The extrinsic semiconductor thus formed is called a *p*-type semiconductor.

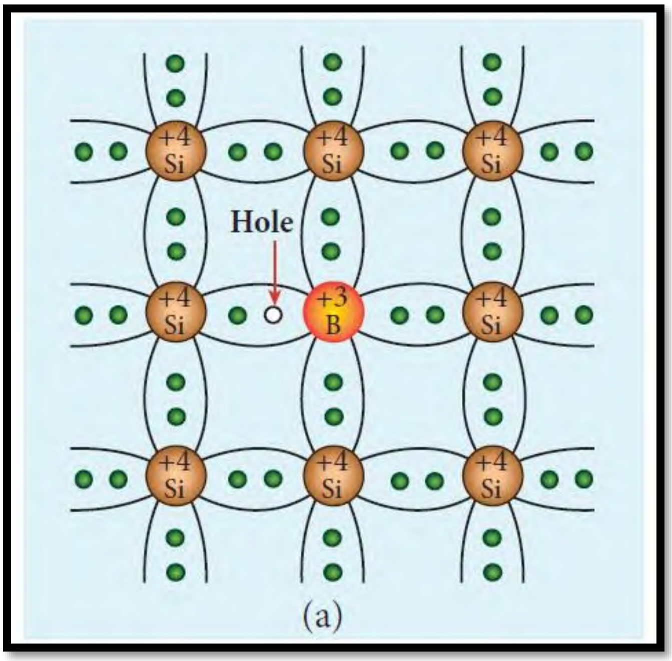

Fig 6 *p*-type extrinsic semiconductor (a) Hole generated by the dopant

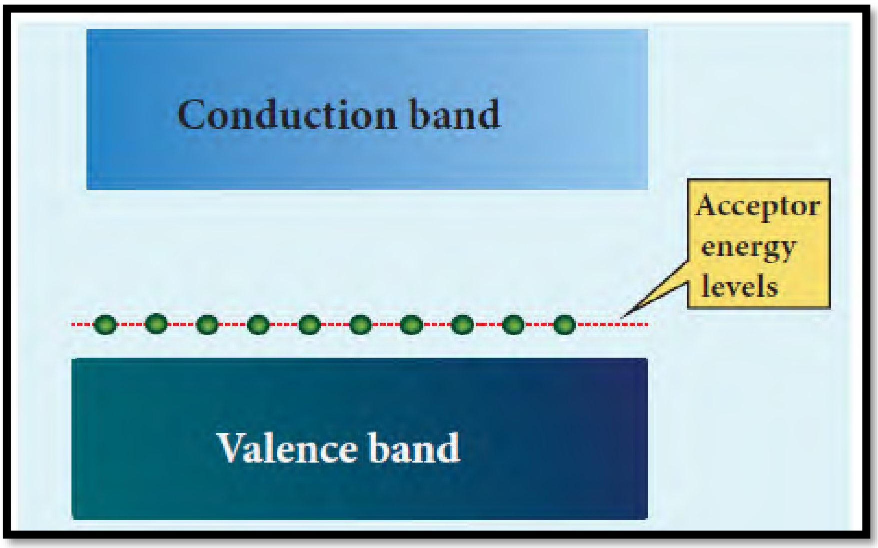

Fig 6: (b) Representation of acceptor energy level.

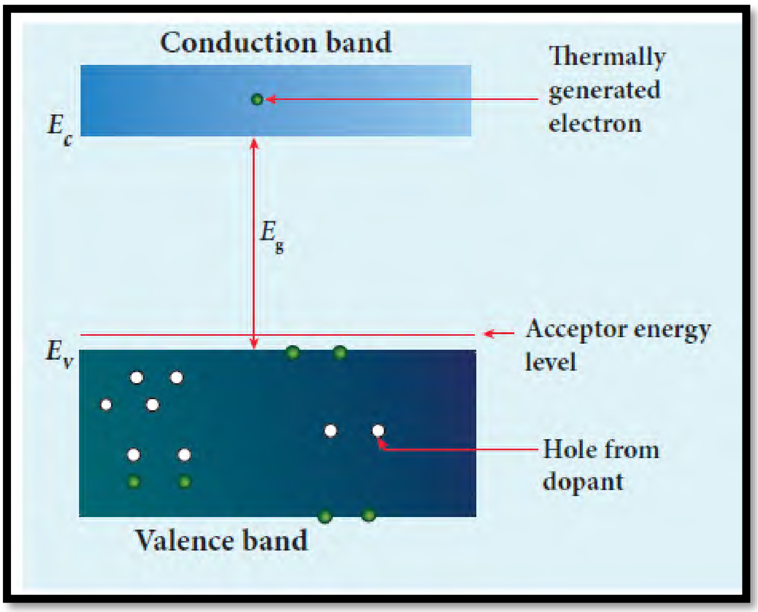

Fig 7: Thermally generated electron in the conduction band and the holes generated by the dopants in the valence band (p-type semiconductor).

EXERCISE QUESTIONS

1. A semiconductor is formed by __________________bonds.

   (A) covalent
   (B) electrovalent
   (C) co-ordinate
   (D) none of the above

2. A semiconductor has __________________temperature coefficient of resistance.

   (A) positive
   (B) zero
   (C) negative
   (D) none of the above

3. The most commonly used semiconductor is __________________

   (A) germanium
   (B) silicon
   (C) carbon
   (D) sulphur

4. A semiconductor has generally __________________valence electrons.

   (A) 2
   (B) 3
   (C) 6
   (D) 4

5. The strength of a semiconductor crystal comes from __________________

   (A) forces between nuclei
   (B) forces between protons
   (C) electron-pair bonds
   (D) none of the above

6. When a pentavalent impurity is added to a pure semiconductor, it becomes ________

   (A) an insulator
   (B) an intrinsic semiconductor
   (C) p-type semiconductor
   (D) n-type semiconductor

7. Addition of pentavalent impurity to a semiconductor creates many ___________.
   (A) free electrons (B) holes
   (C) valence electrons (D) bound electrons

8. A pentavalent impurity has ________________valence electrons.
   (A) 3 (B) 5 (C) 4 (D) 6

9. An n-type semiconductor is _____________.
   (A) positively charged (B) negatively charged
   (C) electrically neutral (D) none of the above

10. A trivalent impurity has ____________valence electrons.
    (A) 4 (B) 5 (C) 6 (D) 3

9.3 DIODES

9.3.1 P-N Junction formation:

(i) Formation of depletion layer

- A single piece of semiconductor crystal is suitably doped such that its one side is *p*-type semiconductor and the other side is *n*-type semiconductor. The contact surface between the two sides is called *p-n* junction.
- Whenever *p-n* junction is formed, some of the free electrons diffuse from the *n*-side to the *p*-side while the holes from the *p*-side to the *n*-side. The diffusion of charge carriers happens due to the fact that the *n*-side has higher electron concentration and the *p*-side has higher hole concentration.
- **The diffusion of the majority charge carriers across the junction gives rise to an electric current, called diffusion current**.
- When an electron leaves the *n*-side, a pentavalent atom in the *n*-side becomes a positive ion. The free electron migrating into *p*-side recombines with a hole present in a trivalent atom near the junction and the trivalent atom becomes a negative ion. Since such ions are bonded to the neighbouring atoms in the crystal lattice, they are unable to move.
- As the diffusion process continues, a layer of positive ions and a layer of negative ions are created on either side of the junction accordingly. **The thin region near** the **junction which is free from charge carriers (free electrons and holes) is called depletion region** (Figure 8).

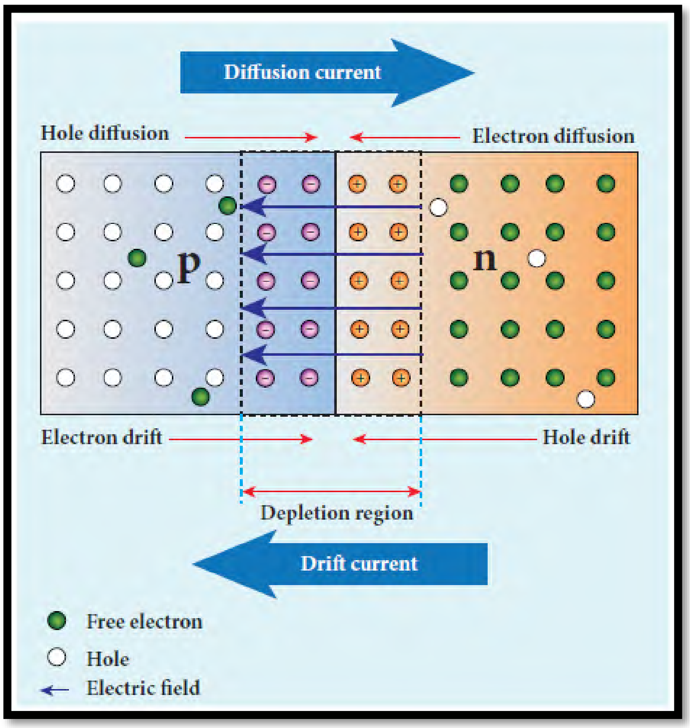

Fig 8 Formation of p–n junction

- An electric field is set up between the positively charged layer in the *n*-side and the negatively charged layer in the *p*-side in the depletion region as shown in the Figure 8. This electric field makes electrons in the *p*-side drift into the *n*-side and the holes in the *n*-side into the *p*-side.
- **The electric current produced due to the motion of the minority charge carriers by the electric field is known as drift current**. The diffusion current and drift current flow in opposite directions.
- Though drift current is less than diffusion current initially, equilibrium is reached between them at a particular time. With each electron (or hole) diffusing across the junction, the strength of the electric field increases thereby increasing the drift current till the two currents become equal.
- Hence at equilibrium, there is no net electric current across the junction. Thus, a *p-n* junction is formed.

(ii) Junction potential or barrier potential

- The movement of charge carriers across the junction takes place only to a certain point beyond which the depletion layer acts like a barrier to further diffusion of free charges across the junction. This is due to the fact that the immobile ions on both sides establish an electric potential difference across the junction.

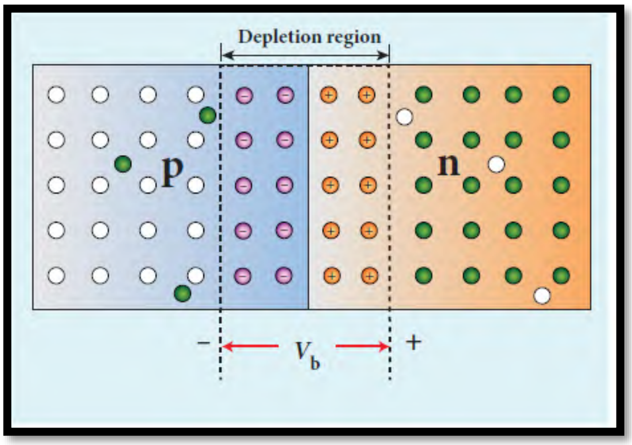

Fig 9: Barrier potential formed across the junction

- **This difference in potential across the depletion layer is called the barrier potential (V_b)** as shown in Figure 9. At 25^0C, this barrier potential is approximately 0.7 V for silicon and 0.3 V for germanium.

9.3.2 P-N Junction diode:

- **A *p-n* junction diode is formed when a *p*-type semiconductor is fused with an *n*-type semiconductor**. It is a device with single *p-n* junction as shown in Figure 10(a) and its circuit symbol is shown in Figure 10(b).

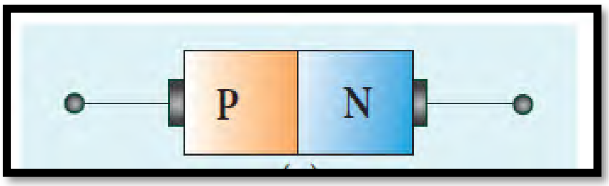

Fig 10: p-n junction diode (a) Schematic representation

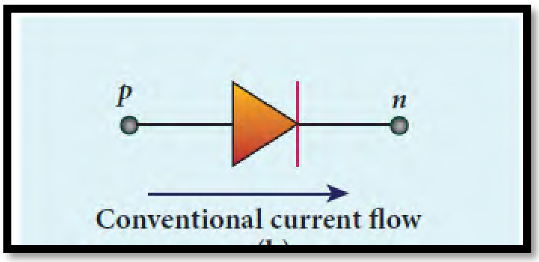

Fig 10 (b) Circuit symbol Biasing a diode

- **Biasing means providing external energy to charge carriers to overcome the barrier potential and make them move in a particular direction.** The charge carriers can either move towards the junction or away from the junction.
- **The external voltage applied to the *p-n* junction is called bias voltage**. Depending on the polarity of the external source to the *p-n* junction, we have two types of biasing:

  (i) Forward bias

  (ii) Reverse bias

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- ***UGTRB***
- ***SGT***
- ***POLYTECHNIC TRB***
- ***BEO***
- ***TET Paper I & II***
- ***College TRB***
- ***Special Teachers***

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PGTRB

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- Tamil
- English
- Mathematics
- Physics
- Chemistry
- Botany
- Zoology
- Economics
- Commerce
- Computer Science
- History

மேற்கண்ட அனைத்து படப்பிரிவுகளுக்கான Study Material-களுடன் Psychology, Tamil Eligibility Book, Question Bank மற்றும் General Knowldge Material-களும் வழங்கப்படும்

TET (Teachers Eligibility Test)

TET தேர்விற்கு நம் Teachers Care Academy-யில் Paper I மற்றும் Paper II என இரண்டு தாள்களுக்கும் பிரத்தியேகமாக பயிற்சிகளை வழங்குகிறோம்

இதற்கு தமிழ்நாடு அரசால் வழங்கப்பட்டுள்ள பள்ளி பாட புத்தகத்தில் இருந்து குறிப்புகளை எடுத்து Study Material-களாக வழங்குகிறோம்

மேலும் Psychology-க்கு TRB-ஆல் வழங்கப்பட்டுள்ள பாடத்திட்டத்தை பின்பற்றி பல்வேறு Reference Book-லிருந்து குறிப்புகளை எடுத்து Study Material-களாக வழங்குகிறோம்

UGTRB

TET தேர்வில் வெற்றி பெற்ற ஆசிரியர்களுக்கு நடத்தப்படும் UGTRB போட்டி தேர்வுக்காக_அனைத்து மொழி பாடத்திற்கும் பயிற்சிகளை வழங்கி வருகிறோம் அதாவது

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- Physics
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- Botany
- Zoology
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SGTRB

TET தேர்வில் வெற்றி பெற்ற ஆசிரியர்களுக்கு நடத்தப்படும் *SGTRB* போட்டி தேர்வுக்காக தமிழ்நாடு அரசால் வழங்கப்பட்டுள்ள பள்ளி பாட புத்தகத்தில் இருந்து குறிப்புகளை எடுத்து *Study Material*-களாக வழங்குகிறோம்

BEO

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College TRB

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Special Teachers

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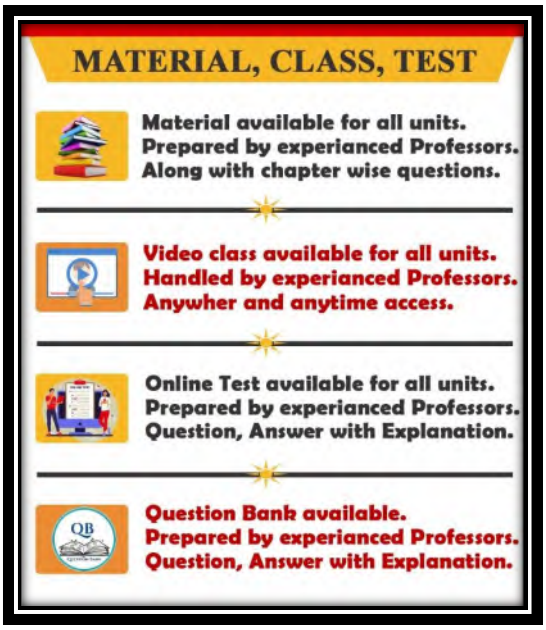

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